2024 The International Mathematical Modeling Challenge (IMMC)

Team Control Number: **2024030**

Abstract

As pet keeping becomes increasingly common, the mechanism of assessing households' readiness for pet ownership plays an important role in promoting the well-being of potential both pets and humans. This report primarily explores the approach to evaluate households' preparation for pet ownership and forecasts future pet ownership.

In the first part of Problem 1, we characterized pets as **domestic animals** primarily kept for companionship or pleasure and constructed a comprehensive cat-ownership evaluation model. First, we pinpointed essential assessment factors such as **time**, **space**, **finances**, **age**, and **prior pet ownership experience**, all contributing to the **readiness score (RS)**. To measure unsuitability, we considered **allergies**, **lifestyle compatibility**, and **commitment to long-term care**, which contribute to the **unsuitability score (US)**. For RS, we employee piecewise linear functions to evaluate each criterion's score and use the **Analytical Hierarchy Process** (AHP) to assign factor weights. US is defined by absolute constraints like allergies and lifestyle or commitment issues, assigned as either 0 or infinitely unsuitable. We then introduced the **evaluation score (ES)** as RS minus US and compared it to a **threshold baseline**. Our model is visualized in a schematic diagram. A household exceeding this threshold is deemed fit for pet ownership, indicating **negligible unsuitability** and a **high readiness score**.

In the second part of Problem 1, we applied our model to diverse case studies, examining **six distinct family scenarios**. Furthermore, we estimated the potential for pet ownership in **New York**, **Shanghai**, and **Shenzhen**, considering variables such as household counts, available living space, and allergy prevalence.

For Problem 2, we adapted our model for four additional pet types: **dogs**, **fish**, **rabbits**, and **birds**, modifying the **weights** of each factor according to the specific needs of these animals. For instance, dogs demand more space and time than fish. To address multi-pet scenarios, we incorporated an **incompatibility variable**, which reflects the potential for discordance between different pet species. This factor is integrated into the unsuitability score (US), allowing us to adjust the initial evaluation score (ES) by subtracting any incompatibility score.

For Problem 3, to quantitatively estimate the future populations and retention rates of pets, we adopted the **logistic model** in household quantity prediction, and assumed a consistency in the **growth rate** of pet adoption suitability. We formulated the total number of pets as a function of the number of households, the suitability for pet ownership and the retention rate. Ultimately, employing Python, we projected the pet population over time, generating a graph that illustrates the **temporal evolution** of pet numbers in Shanghai, Shenzhen and New York in 5, 10 and 15 years.

Finally, we conduct a **sensitivity analysis** on the degree of increase or decrease in the weight of the factors that determine evaluation score (ES). Specifically, we calculate evaluation score for different weight values, resulting in new readiness scores for each family. We also assess our model's **strengths and weaknesses**. Based on our model and analysis, we provide a **one-page recommendation letter** to enhance the synergy between potential pets and human companions.

Keywords: **pet ownership, readiness, unsuitability, AHP, pet demographics**

Contents

1 Introduction

1.1 Background

Pets have played an important role in human society since before recorded history. At least 15,000 years ago, dogs were first domesticated in Central Asia by hunter-gatherers [1]. In ancient times, humans kept pets mostly for their meat, skin, eggs and milk; in medieval Europe, recreational pet keeping grew popular among the nobility, but close association with animals was disapproved by the Christian Church; till the late 18th century, pet ownership became common among the middle classes, being considered as a connection with nature [2]. Nowadays, not only more and more people are keeping their pets, but also the variety of pets increases.

Since the start of the COVID-19 pandemic, 3.2 million households in the UK have got a pet in response to social isolation, once again proving the significance of pet ownership on us [3]. However, some social issues have also arisen. For instance, households' health, economic and social stresses can increase pet abandonment sharply. These abandoned pets, kept and raised in captivity, cannot survive alone. They may experience hunger, thirst, injuries, diseases. They may also spread fast-spreading diseases, posing health risks to both humans and wild lives.

As a result, an assessments on households' suitability to own pets is necessary for the well-being of animals and humans. A questionnaire, which asks for households' financial status, health conditions, etc., is one of the common approaches employed by animal adoption institutions. Similarly, given statistics on families' conditions, either quantitative or qualitative, we will create a mathematical model to determine which households are suitable homes for pets.

1.2 Problem Restatement

The tremendous increment in pet adoption quantity following the COVID-19 pandemic has significantly raised people's concern about pet adoption issues. The International Mission and Maintenance and Care of Animals (IMMC-A) has been demanding a quantitative approach that specializes in the assessment of the suitability of household pet adoption, which helps animal shelters and similar entities to evaluate whether it is appropriate to grant a particular household pet ownership.

In this scenario, we need to find out the input factors that can most directly and indicatively represent the main determinants of household suitability in adopting pets, after which we can develop reasonable mathematical models to procure our conclusion. Specifically, our main tasks are:

- Devise a model to evaluate households' readiness for cat ownership. Take into account 10 input factors. Provide a diagram that shows the model's decision-making process. Apply our model to 6 household examples, 3 qualified cat ownership and 3 unqualified. Use our model to predict current number of households that are prepared for pet ownership in three countries/regions.
- Develop a generalized model from two aspects: pet species and pet quantities. We will incorporate another four species of pets in our model, after which we are going to justify it with 6 specific household examples. We will also consider households that adopt multiple pets, and assess the application of our model to this situation.

- Make an estimation of future pet ownership situation, including analysis of both ownership and retention rate after certain numbers of years, using the same regions and pet species discussed in the preceding problems.
- Compose a one-page letter to IMMC-A to address our solution to the solution to pet adoption evaluation problem and justify it.

2 Assumptions and Justifications

To simplify the problem, we make the following basic assumptions, each of which is properly justified. Additional, more specific assumptions will be introduced as needed to refine our models.

- **Assumption 1: Independence of Household Characteristics:** We assume that the characteristics of each household are independent and not influenced by neighboring households. This means that factors such as space availability, financial resources, and time commitment are considered in isolation for each household.
  Justification: This assumption simplifies the model by avoiding complex inter-household dynamics and dependencies, which would require a more elaborate model to account for neighborhood effects or shared resources.
- **Assumption 2: Stability of Household Attributes:** It is assumed that the attributes of a household remain relatively stable over a short period. This implies that factors like income, family size, and lifestyle do not change significantly in the short term.
  Justification: This assumption is reasonable because significant changes in household attributes typically occur over longer periods. It allows for a snapshot evaluation that is practical for immediate decision-making regarding pet adoption.
- **Assumption 3: Consistency of Scoring Formulas and Thresholds:** The scoring formulas and threshold scores (ES_{thr}) used to determine suitability are consistent across different households and cities.
  Justification: Standardizing these elements ensures that the model is fair and equitable, allowing for comparisons across different households. It also simplifies the model by providing a uniform basis for evaluation.
- **Assumption 4: Generalization of Pet Needs:** We assume that the needs of each type of pet (e.g., cats, dogs, birds, fish, rabbits) can be generalized and represented by specific factors in the model.
  Justification: While individual pets may have unique needs, a generalized approach allows for a scalable and manageable model. It is reasonable as it reflects the typical care requirements for each species, which are well-documented and understood.
- **Assumption 5: Threshold of Suitability:** A specific threshold score (ES_{thr}) is set to determine the suitability of a household for pet ownership. This threshold is considered to be a reliable indicator of a positive environment for the pet.
  Justification: The threshold provides a clear benchmark for decision-making and helps to prevent unsuitable households from adopting pets they cannot adequately care for.

3 Variables and Notations

The symbols outlined in Table 3.1 are commonly used throughout this report. Any additional, other specific symbols will be introduced and explained at the point of their usage.

Table 3.1: **Variables Table**

Variables	Description
S :	Space availability (measured in square meters or square feet)
T :	Time availability (measured in hours per day)
F :	Financial resources (measured in currency, e.g., USD)
E :	Prior pet ownership experience (binary variable: 1 if experienced, 0 otherwise)
A_l :	Allergies (binary variable: 1 if allergies present, 0 otherwise)
L :	Lifestyle compatibility (binary variable: 1 if compatible, 0 otherwise)
A :	Age of household members (measured in years)
C :	Commitment to long-term care (binary variable: 1 if committed, 0 otherwise)
RS :	Readiness score
US :	Unsuitability score
ES :	The overall evaluation score
N_i :	The total number of households in city
Q_i :	The estimated number of households in city i suitable for owning pets

4 Problem 1: Constructing a Comprehensive Cat-Ownership Evaluation Model

In this section, we develop a comprehensive model to evaluate a household's readiness for cat ownership. This model considers a variety of factors, each playing a crucial role in ensuring a healthy and harmonious environment for the cat.

4.1 Defining the Concept of Pet

Before we delve into the assessment of a household's readiness for pet ownership, it is pivotal to establish a clear definition of a pet. Taking into account the vast array of animal species and the cultural differences in pet ownership, we define pets as domesticated animals primarily kept for companionship or pleasure, rather than for work or production purposes. Figure 4.1 lists common pets, including cats, dogs, rabbits, fish, and birds, etc.

4.2 Identifying Key Assessment Criteria

To create a well-rounded model, we identify and elaborate on the following key criteria:

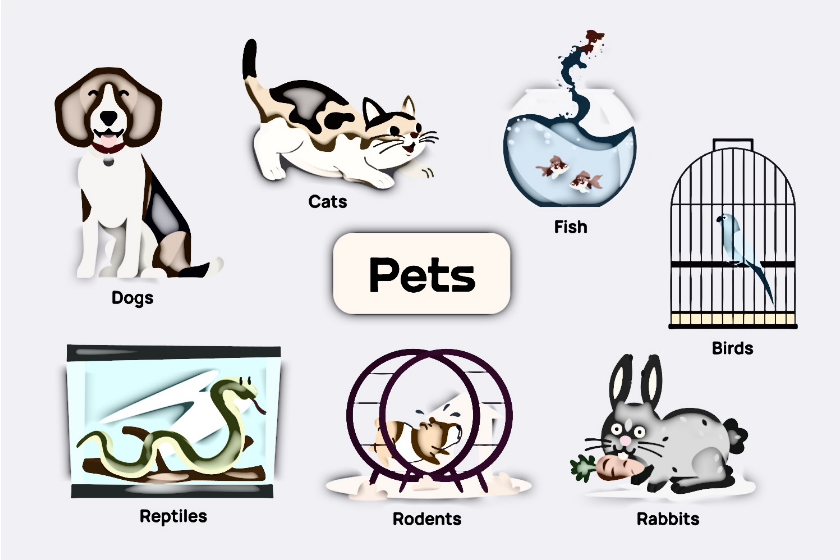


Figure 4.1: Illustrations of various pets.

- **Space Availability (S):** The presence of adequate space can mitigate stress, obesity, and behavioral issues in cats. A spacious environment allows them ample room for physical activities, play, and exploration. While indoor space is instrumental for a cat's safety, an adequate outdoor area can enrich a cat's daily experiences. This is measured in square meters or square feet and should accommodate space for the cat to move, play, and explore without restrictions.

- **Time Availability (T):** Despite their renowned independence, cats require meaningful interaction and care, including feeding, grooming, and playtime. An owner's sufficient time allocation can ensure the fulfillment of a cat's physical and mental needs. The time a potential cat owner can dedicate to their pet is measured in hours per day.

- **Financial Resources (F):** Financial capability, quantified in terms of currency (e.g., USD). The financial implications of owning a cat encompass costs for food, cat litter, veterinary care, toys, and other resources. Financial stability ensures that the cat receives proper care, even in unforeseen circumstances.

- **Prior Pet Ownership Experience (E):** Experience with pet ownership, especially with cats, is a binary variable (1 if experienced, 0 otherwise). Previous experience with pets, particularly with cats, can significantly streamline the process of introducing a new cat to the household.

- **Allergy (Al):** The presence of allergies to cats within the household is a critical binary variable (1 if allergies are present, 0 otherwise). The presence of cat allergies in the household can impact both the residents and the cat's living conditions. It's essential to factor this in to ensure a harmonious living environment for all.

- **Lifestyle Compatibility (L):** Lifestyle compatibility is a binary variable (1 if the household's lifestyle is compatible with cat ownership, 0 otherwise). The household's lifestyle, encompassing work schedules, travel habits, and social activities, influences the level of attention and care a cat receives. A lifestyle compatible with pet ownership can foster a robust bond between the cat and its human family.

- **Age of Household Members (A):** The age of family members influences the nature of care and interaction a cat will receive. Young children may not know how to handle a cat gently, while older individuals may face challenges in providing care. The ages of the household members, measured in years.
- **Commitment to Long-Term Care (C):** The readiness to commit to the cat for its lifetime is a binary variable (1 if the household is committed, 0 otherwise). Given that cats can live for many years, owning a cat is a long-term commitment. This criterion assesses whether the household is prepared to provide consistent care throughout the cat's life.

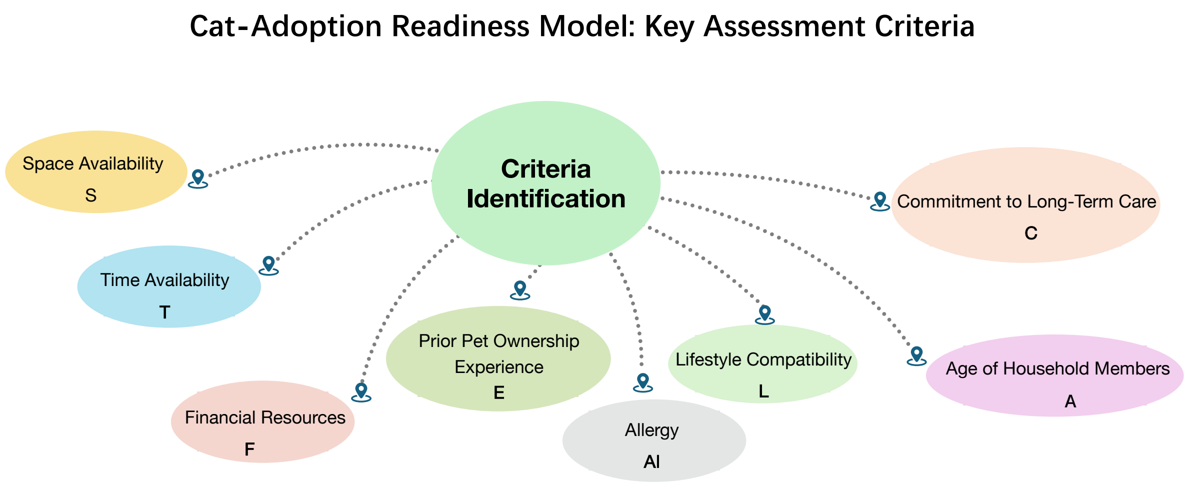


Figure 4.2: Key assessment criteria for pet ownership

Figure 4.2 provides a visual representation of the essential evaluation criteria outlined in our cat-adoption readiness model. To enhance clarity and usability for potential cat owners, we have assigned a specific variable to each factor. Additionally, we have meticulously detailed the measurement process for each criterion to ensure that the assessment can be conducted with ease and precision.

While the model recognizes the existence of additional factors that could influence the suitability of a household for cat adoption, the focus is placed on these primary factors initially. This prioritization allows for a structured approach to the assessment, ensuring that the most significant determinants of successful cat ownership are evaluated first. Subsequent factors can be incorporated as needed to refine the evaluation or address specific concerns within a household.

By concentrating on these pivotal elements, we establish a strong foundation for assessing the readiness for cat adoption. This methodology not only streamlines the evaluation process but also sets clear expectations for prospective cat owners, thereby facilitating a more informed and responsible approach to pet adoption.

4.3 Readiness Modeling for Pet Ownership Score

In developing a readiness model for pet ownership, we construct a scoring system that quantifies a family's suitability for adopting a cat. This scoring system, referred to as the Readiness Score (RS), incorporates a range of factors that are indicative of a family's capability to provide a nurturing and stable environment for a pet. The factors considered in this model are the five: Space (S), Time (T), Financial Resources (F), Age of Household Members (A), and Prior Pet Ownership Experience (E).

Scoring Continuous Variables

For continuous variables such as space, time, financial resources, and the age of household members, we employ a piecewise linear function to calculate their contribution to the Readiness Score (RS). This approach allows for a nuanced assessment that reflects the varying degrees of readiness. For example, the contribution of space to the readiness score (RS_S) is defined as:

$$RS_S = \begin{cases} 0 & \text{if } S < S_{\text{min}} \\ k_s \cdot (S - S_{\text{min}}) & \text{if } S_{\text{min}} \leq S < S_{\text{opt}} \\ RS_{s_{\text{max}}} & \text{if } S \geq S_{\text{opt}} \end{cases} \tag{4.1}$$

- S_{min} is the minimum space deemed necessary for a cat's wellbeing.
- S_{opt} is the optimal space that yields the maximum score for space availability.
- k_S is a scaling factor determining the rate at which the score increases within the acceptable range.
- $RS_{S_{\text{max}}} = 1$ represents the maximal positive impact that space availability can have on the Readiness Score.

Similarly, for time, financial resources, and age of family members (A), we define corresponding piecewise linear functions with appropriate minimum, optimal, and maximum values. The functions are formulated as follows:

$$RS_T = \begin{cases} 0 & \text{if } T < T_{\text{min}} \\ k_T \cdot (T - T_{\text{min}}) & \text{if } T_{\text{min}} \leq T < T_{\text{opt}} \\ RS_{T_{\text{max}}} & \text{if } S \geq T_{\text{opt}} \end{cases} \tag{4.2}$$

$$RS_F = \begin{cases} 0 & \text{if } F < F_{\text{min}} \\ k_F \cdot (F - F_{\text{min}}) & \text{if } S_{\text{min}} \leq F < F_{\text{opt}} \\ RS_{F_{\text{max}}} & \text{if } F \geq F_{\text{opt}} \end{cases} \tag{4.3}$$

For the age of family members (A), we score it as an interval index to account for the varying impact different ages may have on pet care responsibilities:

$$RS_A = \begin{cases} 1 - \frac{A_{\text{min}} - A}{C_A} & \text{if } A < A_{\text{min}} \\ RS_{A_{\text{max}}} & \text{if } A_{\text{min}} \leq A \leq A_{\text{max}} \\ 1 - \frac{A - A_{\text{max}}}{C_A} & \text{if } A > A_{\text{max}} \end{cases} \tag{4.4}$$

where C_A is a constant that moderates the change in score outside the optimal age range, and $RS_{A_{\text{max}}} = 1$ represents the maximum score attainable for the age factor.

Through these mathematical formulations, the Readiness Score becomes a quantifiable metric that can be systematically applied to assess a family's preparedness for pet ownership, ensuring a responsible and suitable match between the cat and its potential new home.

Incorporating Binary Variables into the Readiness Score

Binary variables, such as the presence or absence of previous pet ownership experience, are critical in evaluating a family's readiness for adopting a new pet. These variables can be effectively integrated into the Readiness Score (RS) by assigning fixed scores based on their binary outcomes. For the binary variable representing previous pet ownership experience (E), the scoring function can be expressed as:

$$RS_E = \begin{cases} score_{E0} & \text{if } E = 0 \\ score_{E1} & \text{if } E = 1 \end{cases} \tag{4.5}$$

where:

- $score_{E0}$ represents the score assigned when there is no previous pet ownership experience. This value is typically set to 0, signifying that the lack of experience does not contribute positively to the Readiness Score.
- $score_{E1}$ is the score assigned when there is previous pet ownership experience. This value is set based on the perceived value of past experience in the care and management of pets. For instance, a score of 1 indicates that previous experience is considered beneficial and thus contributes positively to the overall Readiness Score.

By using a binary scoring system for such variables, we ensure that they are factored into the overall assessment without the need for normalization, as they inherently provide a discrete contribution to the total score. This method allows for a straightforward and interpretable integration of binary factors alongside the continuous variables previously discussed.

In the comprehensive evaluation model, each factor, whether continuous or binary, will be given an appropriate weight based on its relative importance in determining pet ownership readiness. The weighted factors are then combined to calculate the final Readiness Score, which serves as a quantitative measure of a family's potential for successfully adopting and caring for a pet.

Weight Calculation Through Analytic Hierarchy Process (AHP)

After establishing the scoring methods for each criterion, we will employ the Analytic Hierarchy Process (AHP) to determine the weights of the five factors: Space (S), Time (T), Financial Resources (F), Age of Household Members (A), and Previous Pet Ownership Experience (E). AHP is a multi-criteria decision-making approach that organizes and analyzes complex decisions by creating a structured model based on mathematics and behavioral science. It quantifies the criteria affecting the decision-making process in a hierarchical order. The following steps outline the process:

1. Establishing the Hierarchical Structure: The initial step is to define the objective of the decision-making process, which, in this case, is to calculate a comprehensive readiness score for cat adoption. We then identify the criteria influencing this decision, which are the five factors mentioned above: Space (S), Time (T), Financial Resources (F), Age of Household Members (A), and Previous Pet Ownership Experience (E). The hierarchical structure is illustrated in Figure 4.3, which shows the goal at the top, followed by the criteria levels.

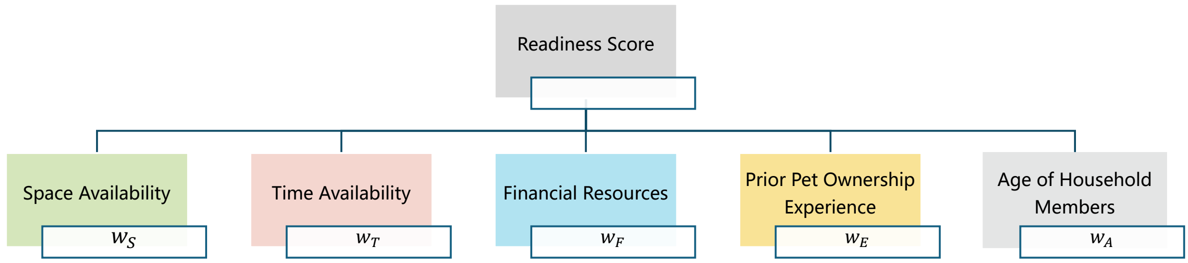


Figure 4.3: The hierarchical structure for Readiness Score.

2. Constructing the Pairwise Comparison Matrix: At the criteria level, we develop a pairwise comparison matrix M, which assesses the relative importance of the factors. This 5×5 matrix M contains elements m_{ij}, each indicating the importance of factor i relative to factor j. We adopt a 1-9 scale for the values of m_{ij}, with 1 indicating equal importance and 9 indicating extreme disparity in importance.

3. Individual Priority Ranking and Consistency Check: We derive the weight vector w_i for each factor by calculating the eigenvector associated with the maximum eigenvalue of the judgment matrix M. A consistency check follows, ensuring that the matrix's judgments are coherently consistent. This involves calculating the Consistency Index (CI) and the Consistency Ratio (CR); a CR value below 0.1 signifies acceptable consistency.

Now we have five factors: Space S, Time T, Financial Resources F, Age of Household Members A, and Experience E. We need to compare the relative importance of these factors. Drawing from relevant literature [1], we assign values for pairwise comparisons as follows:

$$M = \begin{bmatrix} 1 & m_{ST} & m_{SF} & m_{SA} & m_{SE} \\ \frac{1}{m_{ST}} & 1 & m_{TF} & m_{TA} & m_{TE} \\ \frac{1}{m_{SF}} & \frac{1}{m_{TF}} & 1 & m_{FA} & m_{FE} \\ \frac{1}{m_{SA}} & \frac{1}{m_{TA}} & \frac{1}{m_{FA}} & 1 & m_{AE} \\ \frac{1}{m_{SE}} & \frac{1}{m_{TE}} & \frac{1}{m_{FE}} & \frac{1}{m_{AE}} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1/5 & 4 & 1/5 \\ 1/2 & 1 & 1/3 & 3 & 1/3 \\ 5 & 3 & 1 & 7 & 2 \\ 1/4 & 1/3 & 1/7 & 1 & 1/3 \\ 5 & 3 & 1/2 & 3 & 1 \end{bmatrix} \tag{4.6}$$

- Space S is slightly more important than Time T, so $m_{ST} = 2$.
- Space S is less important than Financial Resources F, so $m_{SF} = 1/5$.
- Space S is more important than Age of Household Members A, so $m_{SA} = 4$.
- Space S is less important than Experience E, so $m_{SE} = 1/5$.
- Time T is less important than Financial Resources F, so $m_{TF} = 1/3$.
- Time T is more important than Age of Household Members A, so $m_{TA} = 3$.
- Time T is less important than Experience E, so $m_{TE} = 1/3$.
- Financial Resources F are more important than Age of Household Members A, so $m_{FA} = 7$.
- Financial Resources F are slightly more important than Experience E, so $m_{FE} = 2$.

- Age of Household Members A is less important than Experience E, so $m_{AE} = 1/3$.

Next, we need to calculate the weight vector W and the consistency ratio CR for this judgment matrix. This typically involves the following steps:

1. Calculating the eigenvalues and eigenvectors of the judgment matrix M, identifying the principal eigenvalue as 5.408.
2. Deriving the weight vector W from the principal eigenvector and normalizing it. The normalized eigenvector is

$$\text{weight} = [0.125, 0.103, 0.426, 0.051, 0.295]^\top. \tag{4.7}$$

Figure 4.4 vividly illustrates the distribution of weights among the five criteria, with Financial Resources (F) commanding the highest weight, signifying its paramount importance in the overall readiness assessment for cat adoption.

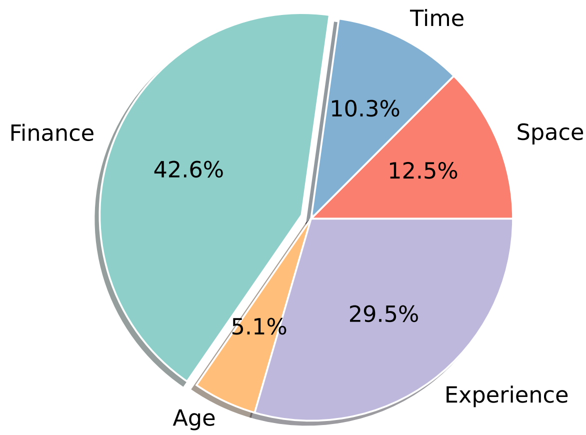


Figure 4.4: Weight for Space S, Time T, Financial Resources F, Age of Household Members A, and Experience E.

3. Computing the Consistency Index CI with the formula $CI = (\lambda_{max} - n)/(n-1)$, where λ_{max} is the principal eigenvalue and n is the matrix's dimension. The calculated CI is 0.147.
4. Retrieving the corresponding Random Consistency Index RI from a standard list (see Table 4.1) based on the matrix's order, which is 1.12 for a 5×5 matrix M.

Table 4.1: **Random Consistency Index (RI)**

Order of the Matrix	1	2	3	4	5	6	7	8	9
RI Value	0.00	0.00	0.58	0.90	1.12	1.24	1.32	1.41	1.45

5. Calculating the Consistency Ratio CR using the formula $\text{CR} = \frac{\text{CI}}{\text{RI}}$. Our calculated $CR = 0.09 < 0.1$ indicates that the judgment matrix's consistency is acceptable.

Comprehensive Readiness Score Calculation

The comprehensive readiness score RS can be calculated through the following weighted sum:

$$RS = w_S \cdot RS_S + w_T \cdot RS_T + w_F \cdot RS_F + w_A \cdot RS_A + w_E \cdot RS_E \tag{4.8}$$

Here, w_S, w_T, w_F, w_A, and w_E are the weights for Space, Time, Financial Resources, Age of Household Members, and Experience, respectively, determined through the AHP. Meanwhile, RS_S, RS_T, RS_F, RS_A, and RS_E are the scores for each factor, calculated according to the previously defined methods.

This method allows us not only to quantify the contribution of each factor but also to weight these contributions according to their relative importance to the success of adoption, thus yielding a comprehensive and quantifiable readiness score. This score can serve as a basis for decision-making in cat adoption, aiding adopters and adoption agencies in making more informed choices.

4.4 Unsuitability Modeling for Pet Ownership

While assessing a family's readiness is crucial in the pet adoption process, **it is equally important to evaluate potential unsuitability factors**. Certain conditions, such as the presence of allergies within the family, can render a household generally unsuitable for pet ownership. It is essential to identify and consider these disqualifying factors to prevent adverse outcomes for both the family and the pet. Therefore, **we introduce an unsuitability assessment to complement the readiness evaluation.**

The Unsuitability Score (US) is designed to encapsulate factors that may have a detrimental impact on the pet adoption process. These factors are deemed non-negotiable and include: Allergies (A_l), Lifestyle Compatibility (L) and Commitment to Long-term Care (C).

These factors serve as strict constraints in the evaluation process. If any of these factors indicate unsuitability (i.e., the family has allergies, an incompatible lifestyle, or a lack of commitment), the Unsuitability Score (US) is triggered. The scoring mechanism is as follows:

$$US = \begin{cases} +\infty & \text{if any unsuitability factor is present} \\ 0 & \text{if all factors indicate suitability} \end{cases} \tag{4.9}$$

A family is considered suitable for pet adoption if and only if the Unsuitability Score is 0 and the Readiness Score (RS) is sufficiently high.

4.5 Comprehensive Evaluation Model for Pet Ownership

To arrive at a final decision regarding pet adoption, we propose a Comprehensive Evaluation Score (ES). This score takes into account both the readiness of the family and any unsuitability factors that may be present. The score is calculated using the following formula:

$$ES = RS - US \tag{4.10}$$

Here, RS represents the Readiness Score derived from the assessment of the family's preparedness, while US is the Unsuitability Score as defined above. The Comprehensive Evaluation Score (ES) effectively integrates both assessments, ensuring that families who are both ready and suitable are given priority in the pet adoption process.

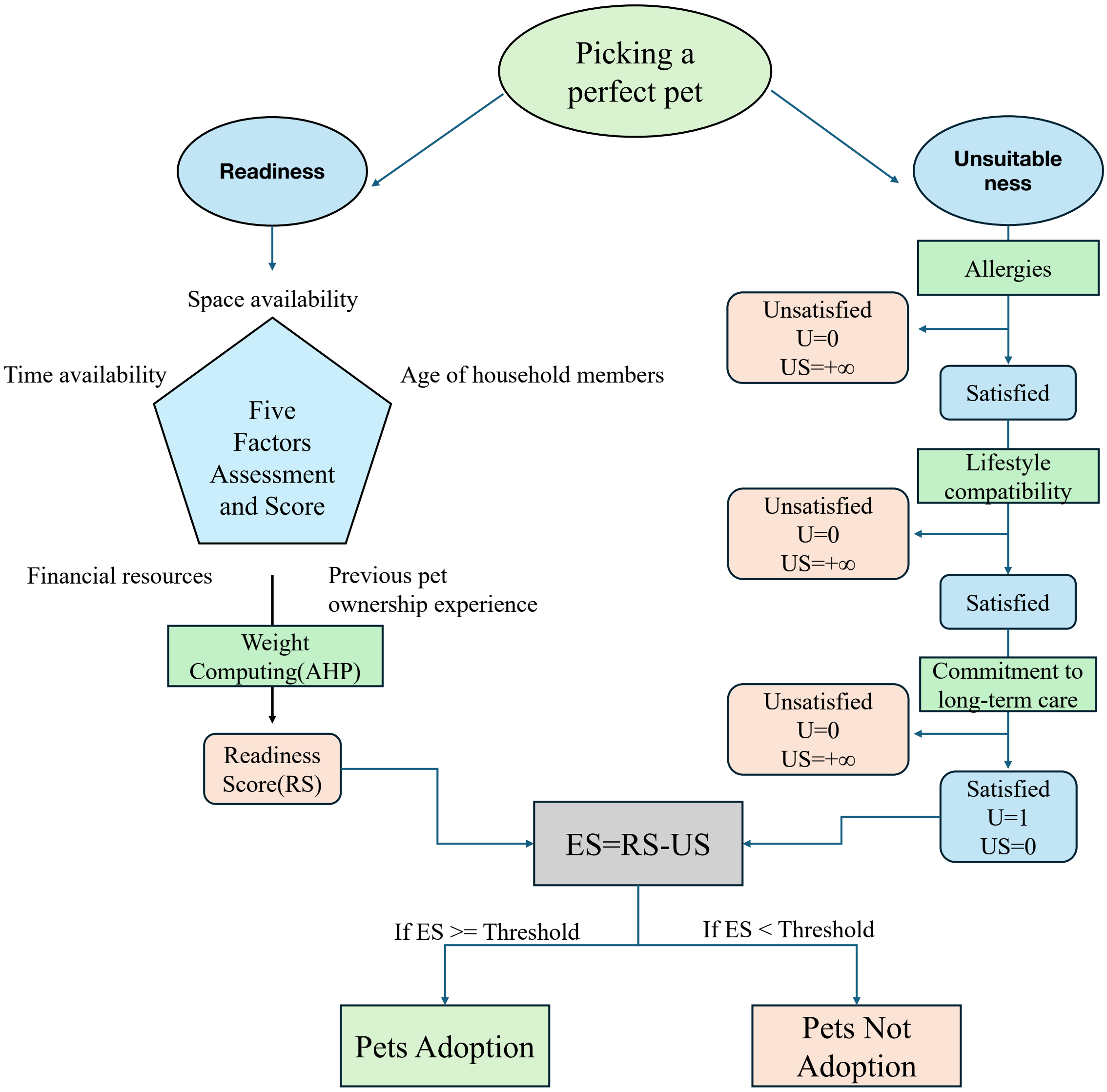


Figure 4.5: The framework of our dual-factor model. A family is considered suitable for pet adoption if and only if the Unsuitability Score is 0 and the Readiness Score is sufficiently high.

Thresholds and Decision Rules

In order to make informed adoption decisions, we set a benchmark Evaluation Score (ES_{thr}). Families are then evaluated based on this threshold using the following rules:

- If $ES \geq ES_{thr}$, the family is considered ready and suitable for adopting a cat.
- If $ES < ES_{thr}$, the family is considered not ready and unsuitable for adopting a cat.

This dual-factor model ensures a balanced assessment by allowing positive aspects to mitigate some negative ones. However, it is designed to recognize that certain significant factors, such as severe allergies or an incompatible living environment, cannot be outweighed by positive aspects. The overall assessment model can be shown in Figure 4.5.

4.6 Six Family Cases Analysis

To demonstrate the application of the comprehensive assessment model for cat, we examine the cases of six families:

1. **Family 1**: This family has a spacious living environment with 120 square meters of space and allocates 2 hours per day for pet care. Their financial resources amount to $30,0000, presumably on a daily or monthly basis, and the average age is 22.5 years old. They have prior experience with pet ownership. There are no allergies to pets within the household, their lifestyle is well-suited for cat ownership, and they are committed to providing long-term care for a pet.

2. **Family 2**: With 100 square meters of space and 1.5 hours available per day for pet care, this family's financial resources are $20,0000. The age of the average age is 42 years, and they have no prior pet ownership experience. Similar to Family 1, they do not have allergies, their lifestyle is conducive to owning a cat, and they are ready to commit to long-term care.

3. **Family 3**: This family lives in an 80 square meter home and dedicates 3 hours per day to pet-related activities. They have a financial budget of $24,0000 and experience with pet ownership. The average age is 35 years old. They do not have allergies, their lifestyle supports cat ownership, and they are prepared for the long-term responsibilities of pet care.

4. **Family 4**: Occupying a 90 square meter space, this family can spend 2 hours daily on pet care and has $25,0000 available for pet expenses. The average age is 32 years old. They have no prior experience with pets and, unlike the first three families, they have allergies to pets. However, their lifestyle is compatible with having a pet, but they lack commitment to long-term care.

5. **Family 5**: This family has 110 square meters of living space and 1 hour per day to care for a pet. They have substantial financial resources at $32,0000 and no pet ownership experience. The average age is 38 years old. There are no allergies present, but their lifestyle is not compatible with cat ownership, although they are willing to provide long-term care if they decide to get a pet.

6. **Family 6**: With the most space at 40 square meters and 3 hours available for pet care, this family has the least financial resources at $18,0000. The average age is 18 years old and they have not previously owned a pet. They do not have allergies and their lifestyle is suitable for pet ownership, and they are committed to long-term care.

Upon closer examination of the suitability of families 4 and 5 for pet ownership, it becomes evident that their profiles present significant obstacles. Their unsuitability scores are infinite, a clear indication that they fail to meet the criteria for at least one of the three critical unsuitability indicators. This renders them fundamentally incompatible with the responsibilities of cat ownership.

In stark contrast, Families 1, 2, and 3 each display a harmonious blend of space, time, financial resources, and a conducive environment for pet ownership. None of these families exhibit any unsuitable indicators, and their calculated readiness scores are 0.89418756, 0.60671606, and 0.79045688, respectively. These scores surpass the threshold of 0.6 that we have established as the benchmark for pet ownership suitability, indicating their aptness for the responsibilities and joys of caring for a pet.

Family 6, despite having ample space and time, falls short with a readiness score of 0.2414134, which is well below the established threshold. This low score is a clear sign of their unpreparedness for pet ownership, as it reflects significant deficiencies in essential areas required for a stable pet-owning environment.

Figure 1 offers a graphical representation of the readiness scores for Families 1, 2, 3, and 6 across the five evaluated factors. The visualization underscores the low scores of Family 6, particularly highlighting the areas in which they are lacking. This visual confirmation supports our analytical conclusion, providing a clear and immediate understanding that Family 6, despite certain favorable conditions, is not currently suited for pet ownership.

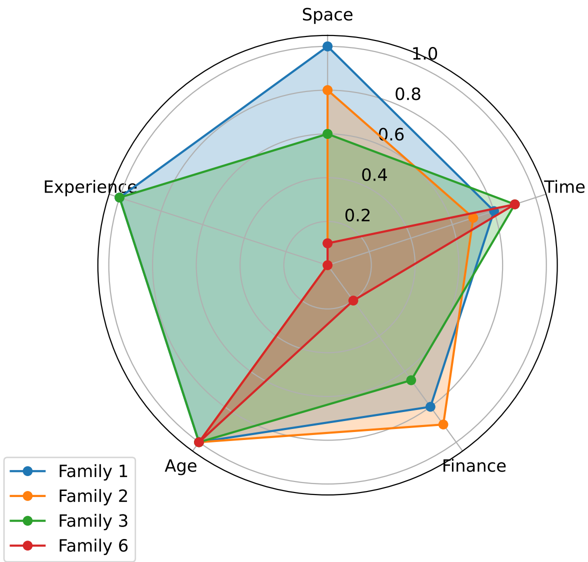


Figure 4.6: The readiness scores for Families 1, 2, 3, and 6 across the five evaluated factors.

4.7 Estimating the Potential Pet-Owning Households

To project the number of households that could potentially own pets, we focus on three distinct urban environments: **Shanghai (China)**, **New York (USA)**, and **Shenzhen (China)**. These cities not only represent the economic and cultural diversity of Asia and North America but also offer a glimpse into varied lifestyles and urban dynamics which could influence pet ownership.

The objective is to estimate the volume of households within these cities that are suitable for cat ownership, under the premise that our comprehensive assessment model is universally applicable.

A model is formulated to estimate the number of households suitable for cat ownership in each city:

$$Q_i = N_i \times P_i \tag{4.11}$$

Here, Q_i represents the estimated number of households in city i suitable for owning cats, N_i is the total number of households in city i and P_i: is the proportion of households in city i

deemed suitable for cat ownership.

To determine P_i, an evaluation of households is required. Using the earlier defined scoring system, we calculate the Evaluation Score for each household and compare it against the threshold score:

$$P_i = \frac{1}{N_i} \sum_{j=1}^{N_i} I(ES_{i,j} \geq ES_{thr}) \tag{4.12}$$

The indicator function $I(x)$ equals 1 if x is true and 0 otherwise.

This proportion accounts for various factors such as financial status, living conditions, demographics, and willingness to commit to pet ownership. We introduce additional variables:

- A_i: Proportion of households in city i with allergies to cats.
- L_i: Proportion of households in city i with lifestyles incompatible with cat ownership.
- C_i: Proportion of households in city i unwilling to commit to long-term pet care.
- E_i: Proportion of households in city i with economic means to afford a cat.
- SA_i: Proportion of households in city i with adequate space for a cat.
- TA_i: Proportion of households in city i with enough time to care for a cat.
- FR_i: Proportion of households in city i with the financial resources for cat care.
- AA_i: Proportion of households in city i with family members aged 18-65, deemed the ideal age range for cat ownership.

Incorporating these factors, our refined model becomes:

$$P_i = (1 - A_i) \times (1 - L_i) \times (1 - C_i) \times E_i \times SA_i \times TA_i \times FR_i \times AA_i \tag{4.13}$$

This formula presupposes that each factor is independent and can be quantified as a ratio. To apply this model, we must gather relevant data, such as household demographics, living space, time availability, financial resources, allergy prevalence, lifestyle compatibility, and commitment to long-term pet care. These data are sourced from surveys, public records, or expert assessments.

Shanghai

Given Shanghai's population of approximately 24.2 million [9], we can estimate the number of households based on the average family size: $N_{\text{Shanghai}} = \frac{24{,}200{,}000}{3} \approx 8,066,667$.

For Shanghai, the variables are estimated as follows: Allergy prevalence: $A_{\text{Shanghai}} = 5\%$, Lifestyle compatibility: $L_{\text{Shanghai}} = 10\%$, Unwillingness to commit: $C_{\text{Shanghai}} = 9\%$, Space availability: $SA_{\text{Shanghai}} = 60\%$, Time availability: $TA_{\text{Shanghai}} = 60\%$, Financial resources: $FR_{\text{Shanghai}} = 70\%$ and Age suitability: $AA_{\text{Shanghai}} = 50\%$. The proportion of households suitable for cat ownership is calculated as:

$$P_{\text{Shanghai}} = (1 - 0.05) \times (1 - 0.1) \times (1 - 0.09) \times 0.60 \times 0.60 \times 0.70 \times 0.50 \approx 9.8\%. \tag{4.14}$$

Hence, the estimated number of suitable households for cat ownership in Shanghai is:

$$Q_{\text{Shanghai}} = N_{\text{Shanghai}} \times P_{\text{Shanghai}} \approx 8,066,667 \times 9.8\% \approx 790,810. \tag{4.15}$$

Shenzhen

With 12.59 million households in Shenzhen [8], the suitability for cat ownership is calculated based on the following factors: Allergy prevalence: 6%, Lifestyle compatibility: 30%, Unwillingness to commit: 10%, Space availability: 60%, Time availability: 70%, Financial resources: 70%, Age suitability: 50%.

The proportion suitable for cat ownership in Shenzhen is:

$$P_{\text{Shenzhen}} = (1 - 0.06) \times (1 - 0.3) \times (1 - 0.1) \times 0.60 \times 0.70 \times 0.70 \times 0.50 \approx 8.7 \quad (4.16)$$

The number of suitable households in Shenzhen is:

$$Q_{\text{Shenzhen}} = N_{\text{Shenzhen}} \times P_{\text{Shenzhen}} = 12590000/3 \times 8.7\% \approx 594,000. \quad (4.17)$$

New York

For New York's 8.33 million population, the suitability for cat ownership is determined by: Allergy prevalence: 7%, Lifestyle compatibility: 7%, Willingness to commit: 7%, Space availability: 50%, Time availability: 50%, Financial resources: 65%, Age suitability: 75%.

The proportion of households suitable for cat ownership in New York is:

$$P_{\text{New York}} = (1 - 0.07)^3 \times 0.50^2 \times 0.65 \times 0.75 \approx 9.9\%$$

The estimated number of suitable households for cat ownership in New York is:

$$Q_{\text{New York}} = N_{\text{New York}} \times P_{\text{New York}} \approx 8330000/3 \times 9.9\% \approx 275,828.$$

5 Problem 2: Adapting the Model for Various Pets

The versatility of our model allows for its application to a variety of household pets. To accommodate different species, we can introduce additional parameters that account for the unique care requirements, family preferences, and the compatibility of the pet with the household environment. However, to maintain the model's simplicity and effectiveness, we will cap the number of input factors at nine.

5.1 Model Adaptation for Dog, Bird, Fish and Rabbit

For each type of pet—dogs, birds, fish, and rabbits—we will apply the established Readiness Score (RS) and Unsuitability Score (US) framework, making species-specific modifications. These adjustments will reflect the unique needs of each pet. For instance, when considering dogs, factors such as the need for ample space for exercise and time for regular walks become crucial. The weight coefficients assigned to each input factor will be recalibrated to reflect the priorities and necessities of each pet species.

Our current weight distribution for cats is [0.125, 0.103, 0.426, 0.051, 0.295]. To tailor the weights for dogs, fish, rabbits, and birds, we need to consider the distinctive requirements of each:

- **Dogs:** Require more space and exercise, and demand more time for walks and companionship. The weight for space and time availability should be increased: [0.21, 0.09, 0.39, 0.05, 0.26]

- **Fish:** Require less space and time but need specific water conditions and tank maintenance. The weights for space and time can be reduced, while the importance of financial resources for maintaining an aquarium could be increased: [0.08, 0.05, 0.53, 0.05, 0.29].
- **Rabbits:** Require less space than dogs but more than cats and have specific dietary and exercise needs. The weight for space can be moderately high, with additional consideration for the financial and time resources for their care: [0.16, 0.09, 0.44, 0.05, 0.26].
- **Birds:** Require less space for living but need engagement and a suitable environment to thrive. The weight for space can be lower, but the emphasis on the household's ability to provide a stimulating environment should be considered: [0.08, 0.1 , 0.48, 0.05, 0.29]

Our model can be expressed in the formula:

$$RS = w_S \cdot RS_S + w_T \cdot RS_T + w_F \cdot RS_F + w_A \cdot RS_A + w_E \cdot RS_E \tag{5.1}$$

We will assign new weight coefficients for each pet species to align with these considerations, ensuring the model remains robust across different scenarios. Finally, we can get the overall evaluation score:

$$ES_{pet} = RS_{pet} - US_{pet} \tag{5.2}$$

where S_{pet} indicates the suitability score of the considered species, and so follows RS_{pet} and $USpet$ as its readiness score and unsuitableness score, respectively.

For each pet species, we will set up a threshold value, marked by $S_{pet,thr}$, that conforms to the following rules in judging:

- If $S_{pet} \geq S_{pet,thr}$, then the family is deemed prepared for adopting the pet.
- Otherwise, the family is deemed unqualified for adopting the pet.

This methodology allows us to generate individualized and concentrated assessment of each pet species, while maintaining the overall generality and expandability of the model.

Six Family Cases Analysis

To help deliver the extended model in a more articulated and convincing manner, we will once again choose six households to analyze. We will consider every family's suitability score on each of the five chosen pets (cats, dogs, birds, fish and rabbits), and determine whether it is appropriate for a certain family to adopt a particular pet species based on the threshold value.

From the original six families, we have identified that families 4 and 5 are not viable candidates for pet ownership due to allergies or lifestyle conflicts. Our attention, therefore, shifts to the suitability assessment of families 1, 2, 3, and 6. As depicted in Figure 5.1, we have computed the Readiness Scores for these households, applying distinct weightings for each pet type.

The analysis reveals that Family 6 is unsuitable for pet ownership across the board, constrained by limited space and financial resources. Family 2's scores hover around the threshold of 0.6, indicating a marginal suitability. Interestingly, Family 2 exhibits a higher propensity for accommodating pets such as dogs, fish, or rabbits, in contrast to cats and birds, when considering their specific needs and the family's capacity to meet them.

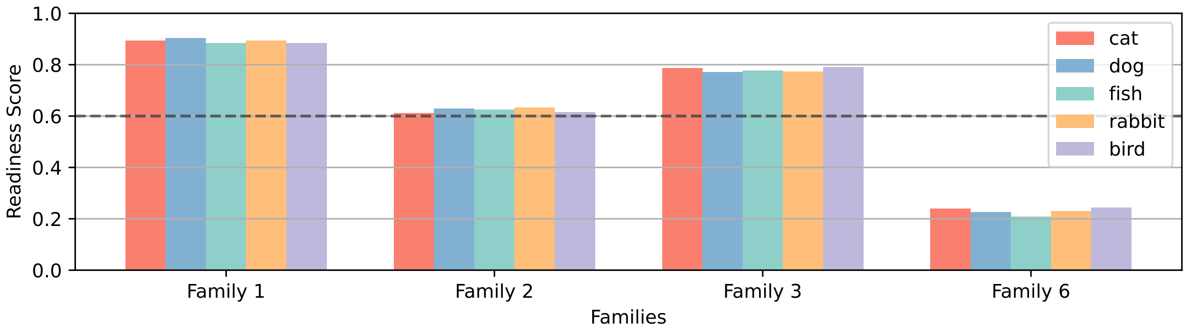


Figure 5.1: Readiness Scores for Pet Adoption in Four Households

5.2 Model Involving Multiple Pet Adoptions

Under the circumstances of adopting multiple pets, the one essential factor to be taken into account is the compatibility between pets, which reflects whether the new pets can coexist peacefully with the original pets; or if multiple pets are simultaneously wanted, whether they can live harmoniously with each other.

The originally existing pets may impinge on the integration of new cats into the family. An Appropriate introduction of pets based on compatibility is crucial for preventing the pressure and hostility between the pets.

We can take compatibility between pets as a input factor of the unsuitability score (US), which can be expressed as:

$$US_{compatibility} = f(C) \tag{5.3}$$

where C represents the compatibility between pets and f is a function used to map compatibility into unsuitability score. If two pets employ a relatively high compatibility (two cats, for instance), the $US_{compatibility}$ will be low; on the contrary, pets that tend to reject each other's presence, may have a higher $US_{compatibility}$ value.

Hence our model can be modified as follows:

$$S_{pet} = RS_{pet} - US_{pet} - US_{compatibility} \tag{5.4}$$

We will again establish a threshold value, marked by $S_{pet,thr}$, that conforms to the following rules in judging:

- If $S_{pet} \geq S_{pet,thr}$, then the family is deemed prepared for adopting the pet.
- Otherwise, the family is deemed unqualified for adopting the pet.

This methodology incorporates consideration of multiple pet adoptions, a common phenomenon in families, while maintaining the overall generality and expandability of the model.

6 Problem 3: Predicting Future Pet Demographics

To forecast future pet demographics, a thorough analysis involving logistic growth models and retention rates is essential. This analysis will help us understand the dynamic changes in pet ownership over time, taking into account various influencing factors such as population growth, urban development, and societal trends.

Defining Key Variables

Let's define the key variables that will be used in our predictive model:

- $N_{i,t}$: The number of households in city i at year t.
- $P_{i,t}$: The proportion of households in city i at year t that are suitable for pet ownership.
- $R_{i,t}$: The retention rate of pets in city i at year t, representing the likelihood of pets staying with their original household over time.
- $Q_{i,t}$: The number of households with pets in city i at year t.
- K_i: The carrying capacity, or the potential maximum number of households in city i.
- r_i: The intrinsic growth rate of the number of households in city i.

6.1 Logistic Growth Model for Population Projection

To project the population of households, we utilize a logistic growth model. The logistic model is particularly useful for modeling populations where there is a natural limit to growth, known as the carrying capacity (K_i). The differential equation for this model is:

$$\frac{dN_i}{dt} = r_i N_i \left(1 - \frac{N_i}{K_i}\right) \tag{6.1}$$

The discrete form of this equation, suitable for annual predictions, is:

$$N_{i,t+1} = N_{i,t} + r_i \times N_{i,t} \times \left(1 - \frac{N_{i,t}}{K_i}\right). \tag{6.2}$$

This equation predicts the number of households in the following year ($N_{i,t+1}$) based on the current year's households ($N_{i,t}$) and adjusts for the growth rate and the approach to the carrying capacity.

Predicting the Proportion Suitable for Pet Ownership

To predict the proportion of households suitable for pet ownership, we must consider factors such as economic growth, cultural shifts, and changes in living conditions. If we assume that the financial suitability for pet ownership increases by 0.01% per year, we can model $P_{i,t}$ as:

$$P_{i,t+1} = P_{i,t} \times (1 + 0.0001)^t \tag{6.3}$$

This equation reflects a compound growth scenario where each year, the proportion increases slightly, representing a gradual increase in the number of households that can afford to own a pet.

6.2 Predicting Total Pet Numbers

To estimate the total number of pets in city i at year t, we combine the predicted number of households, the proportion suitable for pet ownership, and the retention rate:

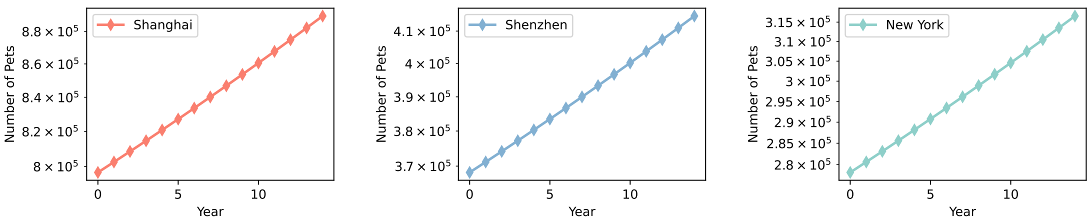
$$Q_{i,t} = N_{i,t} \times P_{i,t} \times R_{i,t} \tag{6.4}$$

This model assumes that the pet retention rate remains constant or changes at a predictable rate. The total number of pets is a function of the number of households, the suitability for pet ownership, and the retention rate.

Long-term Predictions

Using the models outlined above, we can project the pet demographics for intervals of five, ten, and fifteen years into the future, as shown in Figure 6.1. These predictions will help city planners, pet industry stakeholders, and animal welfare organizations make informed decisions regarding infrastructure, services, and regulations.

By incorporating these models into a computational framework, we can simulate various scenarios and perform sensitivity analyses to understand the impact of different factors on future pet demographics. This approach allows for a more nuanced and informed perspective on the evolution of pet ownership trends.



(a) Shanghai (b) Shenzhen (c) New York

Figure 6.1: The pet demographics prediction for intervals of five, ten, and fifteen years into the future.

7 Sensitivity Analysis

We define the readiness score (R_i) for each family i as a linear combination of the five factors: Space (S_i), Time (T_i), Financial Resources (F_i), Age of Household Members (A_i), and Previous Pet Ownership Experience (E_i). The baseline weights ($\boldsymbol{w}^{(0)}$) are given as follows:

$$\boldsymbol{w}^{(0)} = [w_S^{(0)}, w_T^{(0)}, w_F^{(0)}, w_A^{(0)}, w_E^{(0))}] = [0.125, 0.103, 0.426, 0.051, 0.295] \tag{7.1}$$

The readiness score for family i under baseline conditions is therefore:

$$RS_i^{(0)} = 0.125 \cdot RS_{S_i} + 0.103 \cdot RS_{T_i} + 0.426 \cdot RS_{F_i} + 0.051 \cdot RS_{A_i} + 0.295 \cdot RS_{E_i} \tag{7.2}$$

To conduct a sensitivity analysis, we perturb each weight by a specified percentage. Let's denote the perturbation factor as α, which we will set at ±20% for this analysis. This means for each weight w_j, where $j \in \{S, T, F, A, E\}$, we will consider the new weights (w'_j) as:

$$w'_j = w_j^{(0)} \cdot (1 \pm \alpha)$$

Hence, each weight will be considered at two perturbed values, one at 20% less than the baseline and one at 20% more than the baseline. For example, the perturbed weights for Space (w'_S) will be: $w'_S = 0.125 \cdot [0.80, 1.20] = [0.100, 0.150]$.

The sensitivity analysis will be performed by calculating the readiness scores (R'_i) for each family using the perturbed weights. We will start by varying one weight at a time (One-at-a-Time, OAT method) and then proceed to vary multiple weights simultaneously to observe the combined effects.

For the OAT method, we calculate the readiness score for family i with the perturbed weight for Space (w'_S) as follows:

$$R'_i(w'_S) = w'_S \cdot S_i + w_T^{(0)} \cdot T_i + w_F^{(0)} \cdot F_i + w_A^{(0)} \cdot A_i + w_E^{(0)} \cdot E_i. \tag{7.3}$$

This calculation is repeated for each weight and for each perturbation value, resulting in a total of ten new readiness scores for each family (2 perturbations per weight $\times$ 5 weights).

8 Strengths and Weaknesses

8.1 Strengths

- **Application of the Analytic Hierarchy Process**

  By comparing the importance between each other, we construct a series of judgement matrix. After we calculate the eigenvector, we calculate the consistency ratio to check whether it can avoid subjectivity. This results in quantitative readiness score that assist adoption agencies or individuals to make wiser choices.

- **Consideration of unsuitable factors**

  For unsuitable factors, we determine it to be 0 or 1 choices. This means there are some hard constraints which once a family doesn't fit, they are not ready for animal adoption. And this avoids one family adopting pets that may cause lots of serious problems even though they are ready for it.

- **Inclusion of Sensitivity Analysis**

  We carry out a sensitivity analysis, recalculating outcomes under alternative assumptions. These recalculations provide crucial insights into the behavior of our model. Such insights enable us to adapt our model to a variety of situations, making it applicable to real-world conditions in the future.

8.2 Weaknesses

- **Subjective assumption**

  During the comparison of the different values, there are subjective judgements during the process, because we judge the importance of each factor by ourselves. However, different people will have different evaluation criteria.

- **Rough estimation of the ratio that meets the condition**

  When we calculate the ratio or rate of different cities (Shenzhen, Shanghai and New York), we estimate the data by ourselves. Although it is difficult to be accurate, we search for authoritative data on the Internet and make comparisons to decide the final value we use to calculate to make it more reliable.

9 One-page Recommendation Letter

Enhancing the Synergy Between Potential Pets and Human Companions

Dear IMMC-A Directors,

We are writing to you to outline a recommendation that is not only humane and compassionate but also grounded in a mathematical model that ensures the well-being of both domesticated animals and their prospective human families. Our solution, developed through thorough analysis and modeling, proposes a systematic approach to matching pets with humans that is designed to minimize pet abandonment and maximize the mutual benefits of pet ownership.

Our model underscores the criticality of congruence between a pet's requirements and a family's capacity to fulfill them. This harmony is quantified through a comprehensive readiness score, derived from variables such as the family's living space, financial stability, availability of time, the ages of family members, and prior experience with pets. Moreover, specific conditions, such as prevalent allergies within a household, may inherently disqualify a family from pet ownership. Acknowledging and integrating these disqualifying elements is vital to forestall negative repercussions for both the family and the pet. Thus, we introduce an unsuitability assessment to augment the readiness evaluation, ensuring a holistic approach to the adoption process.

The implementation of this user-friendly model in pet adoption processes worldwide can lead to a significant positive change. It would ensure that pets are placed in homes where they can thrive, thereby reducing the likelihood of pets being returned to shelters. Here are the key details of the recommendation:

1. **Adoption Suitability Assessment:** Potential pet owners should undergo an evaluation based on our model to determine their Suitability Score. This will help identify the most compatible pet for their household.
2. **Education and Awareness:** Informing prospective pet owners about the responsibilities of pet care and the long-term commitment involved can prevent impulsive adoptions that may result in abandonment.
3. **Post-Adoption Support:** Offering follow-up support and resources to new pet owners can ease the transition and address any challenges that arise, further reducing the risk of abandonment.

This model is not a mere theoretical construct; it is a call to action. By adopting a quantitative and humane approach to pet adoption, we can create a world where every pet has a home, and every home enjoys the fulfillment that a suitable pet brings.

We are confident that with your leadership, the IMMC-A can advocate for and implement this model, setting a global standard for responsible pet ownership and adoption. We look forward to the possibility of discussing this further and are available for any questions or additional information you may require.

Thank you for considering this recommendation, which holds the promise of a brighter future for countless animals and families worldwide.

Best regards,
2024030

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[9] https://en.wikipedia.org/wiki/Shanghai

[10] https://en.wikipedia.org/wiki/New_York_City

Appendices

Code

```
import matplotlib.pyplot as plt
import matplotlib_inline.backend_inline

def use_svg_display():
    matplotlib_inline.backend_inline.set_matplotlib_formats('svg')
def set_figsize(figsize=(3.5, 2.5), fontsize=10):
    use_svg_display()
    plt.rcParams['figure.figsize'] = figsize
    plt.rcParams['font.size'] = fontsize
set_figsize()

import numpy as np
A = np.array([[1, 2, 1/5, 4, 1/5],
              [1/2, 1, 1/3, 3, 1/3],
              [5, 3, 1, 7, 2],
              [1/4, 1/3, 1/7, 1, 1/3],
              [5, 3, 1/2, 3, 1]])
# eigenvalues, eigenvectors
lamb, v = np.linalg.eig(A)
lamb, v

lambda_max = max(abs(lamb))
loc = np.where(lamb==lambda_max)
loc[0][0]

weight = abs(v[:, np.argmax(lamb)])
weight = weight / sum(weight)
print(weight)

RI_list = [0,0,0.58,0.9,1.12,1.24,1.32,1.41,1.45]
RI = RI_list[len(A)-1]
CI = (lambda_max-len(A))/(len(A)-1)
CR = CI/RI
CR

print('lambda_max=',lambda_max)
print('w=',weight)
print('CI=',CI)
print('RI=',RI)
print('CR=',CR)

np.around(weight, 3)

import matplotlib.pyplot as plt

import seaborn as sns

default_colors = ['#FA7F6F', '#82B0D2', '#8ECFC9', '#FFBE7A', '#BEB8DC', '#
                                        E7DAD2', '#999999']
# print(default_colors)
```

```
set_figsize((3.5*2, 2.5*2), 12)
soldNums = np.around(weight, 3)
kinds = ['Space','Time','Finance','Age', 'Experience']
explode=[0,0,0.05,0,0]
plt.pie(soldNums,
       labels=kinds,
       autopct="%3.1f%%",
       startangle=0,
       colors=default_colors[:len(soldNums)],
       explode=explode,
       shadow=True,
       radius=1,
       wedgeprops=dict(width=1, edgecolor = 'w')
       )
plt.savefig('pie-weight.pdf', bbox_inches='tight')
# plt.savefig('name.svg', bbox_inches='tight')
plt.show()

family1 = {'Space':120,'Time':2,'Finance':30,'Age':22.5, 'Experience':1}
family2 = {'Space':100,'Time':1.5,'Finance':35,'Age':42, 'Experience':0}
family3 = {'Space':80,'Time':3,'Finance':24,'Age':35, 'Experience':1}

family4 = {'Space':90,'Time':2,'Finance':25,'Age':32, 'Experience':0}
family5 = {'Space':110,'Time':1,'Finance':32,'Age':38, 'Experience':0}
family6 = {'Space':40,'Time':3,'Finance':18,'Age':18, 'Experience':0}

scores = np.array([[1, 0.8, 0.8, 1, 1],
          [0.8, 0.7, 0.9, 1, 0],
          [0.6, 0.9, 0.65, 1, 1],
          [0.1, 0.9, 0.2, 1, 0]])
# scores = [[1, 0.8, 0.8, 1, 1]]
scores @ weight

def plot_radar_chart(data, categories, labels=None, title=None):

    data (numpy.ndarray):
    categories (list):
    title (str):
    angles = np.linspace(0, 2 * np.pi, len(categories), endpoint=False)
    angles = np.concatenate((angles, [angles[0]]))

    fig, ax = plt.subplots(figsize=(6, 6), subplot_kw={'polar': True})
    ax.set_theta_offset(np.pi / 2)
    ax.set_theta_direction(-1)

    for i, row in enumerate(data):
        values = np.concatenate((row, [row[0]]))
        ax.plot(angles, values, label=labels[i], color=f'C{i}', marker='o')
        ax.fill(angles, values, alpha=0.25, color=f'C{i}')

    ax.set_xticks(angles[:-1])
    ax.set_xticklabels(categories)
    ax.yaxis.grid(True)
    ax.legend(loc='upper right', bbox_to_anchor=(0.1, 0.1))

    if title:
        plt.title(title)
```

```
    # data = np.array([[80, 90, 65, 60, 85], [95, 60, 75, 70, 60]])
categories = ['Space', 'Time', 'Finance', 'Age', 'Experience']
labels = ['Family 1', 'Family 2', 'Family 3', 'Family 6']
plot_radar_chart(scores, categories, labels)
plt.savefig('family-Q1.pdf', bbox_inches='tight')
# plt.savefig('family.svg', bbox_inches='tight')
plt.show()

weight = np.around(weight, 2)
weight

weight_dog = np.array([weight[0] + 0.1, weight[1], weight[2], weight[3],
                                      weight[4]])
weight_dog = weight_dog / sum(weight_dog)
np.around(weight_dog, 2)

weight_fish = np.array([weight[0]-0.05, weight[1]-0.05, weight[2]+0.1,
                                      weight[3], weight[4]])
weight_fish = weight_fish / sum(weight_fish)
np.around(weight_fish, 2)

weight_rabbit = np.array([weight[0]+0.05, weight[1], weight[2]+0.05, weight
                                      [3], weight[4]])
weight_rabbit = weight_rabbit / sum(weight_rabbit)
np.around(weight_rabbit, 2)

weight_bird = np.array([weight[0]-0.05, weight[1], weight[2]+0.05, weight[3
                                      ], weight[4]])
weight_bird = weight_bird / sum(weight_bird)
np.around(weight_bird, 2)

scores = np.array([[1, 0.8, 0.8, 1, 1],
          [0.8, 0.7, 0.9, 1, 0],
          [0.6, 0.9, 0.65, 1, 1],
          [0.1, 0.9, 0.2, 1, 0]])
scores_cat = scores @ weight
scores_dog = scores @ weight_dog
scores_fish = scores @ weight_fish
scores_rabbit = scores @ weight_rabbit
scores_bird = scores @ weight_bird

y, y1

import matplotlib.pyplot as plt
set_figsize((3.5*3, 2.5))
# some simple data
y = scores_cat
x = np.arange(len(y))
y1 = scores_dog
y2 = scores_fish
y3 = scores_rabbit
y4 = scores_bird
colors = ['#FA7F6F', '#82B0D2', '#8ECFC9', '#FFBE7A', '#BEB8DC', '#E7DAD2',
                                       '#999999']
bar_width = 0.15
tick_label = ['Family 1', 'Family 2', 'Family 3', 'Family 6']
plt.bar(x, y, bar_width, align='center',color=colors[0], tick_label =
                                      tick_label,label = 'cat')
```

```
plt.bar(x + bar_width, y1, bar_width, align='center',color=colors[1],
                                     tick_label = tick_label,label = 'dog'
                                     )

plt.bar(x + 2*bar_width, y2, bar_width, align='center',color=colors[2],
                                     tick_label = tick_label,label = 'fish
                                     ')

plt.bar(x + 3*bar_width, y3, bar_width, align='center',color=colors[3],
                                     tick_label = tick_label,label = '
                                     rabbit')

plt.bar(x + 4*bar_width, y4, bar_width, align='center',color=colors[4],
                                     tick_label = tick_label,label = 'bird
                                     ')

plt.xlabel('Families')
plt.ylabel('Readiness Score')

plt.xticks(x+2*bar_width, tick_label)
plt.grid(axis='y')
plt.legend()
plt.hlines = plt.axhline(y=0.6, color='k', linestyle='--', alpha=0.5)
plt.ylim(0.0, 1)
# plt.yscale('log')
plt.savefig('families-Q2.pdf', bbox_inches='tight')
plt.show()

ratio_shanghai = (1 - 0.1) * (1 - 0.09) * (1 - 0.05) * 0.60 * 0.60 * 0.70 *
                                      0.50
pop_shanghai = 24200000 / 3
pet_shanghai = pop_shanghai * ratio_shanghai
pop_shanghai, pet_shanghai

ratio_shenzhen = (1 - 0.06) * (1 - 0.3) * (1 - 0.1) * 0.60 * 0.70 * 0.70 *
                                     0.50
pop_shenzhen = 12590000 / 3
pet_shenzhen = pop_shenzhen * ratio_shenzhen
ratio_shenzhen, pop_shenzhen, pet_shenzhen

ratio_newyork = (1 - 0.07) * (1 - 0.07) * (1 - 0.07) * 0.50 * 0.50 * 0.76 *
                                      0.65
pop_newyork = 8330000 / 3
pet_newyork = pop_newyork * ratio_newyork
ratio_newyork, pop_newyork, pet_newyork

shanghai_pred = []
K = 30000000
for i in range(15):
    # shanghai_pred.append(pop_shanghai)
    pop_shanghai = pop_shanghai + 0.01 * pop_shanghai * (1 - pop_shanghai /
                                          K)
    ratio_shanghai = ratio_shanghai * (1 + 0.0001) ** i
    pet_shanghai = pop_shanghai * ratio_shanghai * 1
    shanghai_pred.append(pet_shanghai)

    shanghai_pred
```

```
    shenzhen_pred = []
K = 20000000
for i in range(15):
    # shanghai_pred.append(pop_shanghai)
    pop_shenzhen = pop_shenzhen + 0.01 * pop_shenzhen * (1 - pop_shenzhen /
                                         K)
    ratio_shenzhen = ratio_shenzhen * (1 + 0.0001) ** i
    pet_shenzhen = pop_shenzhen * ratio_shenzhen * 1
    shenzhen_pred.append(pet_shenzhen)

    shenzhen_pred

    newyork_pred = []
K = 20000000
for i in range(15):
    # shanghai_pred.append(pop_shanghai)
    pop_newyork = pop_newyork + 0.01 * pop_newyork * (1 - pop_newyork / K)
    ratio_newyork = ratio_newyork * (1 + 0.0001) ** i
    pet_newyork = pop_newyork * ratio_newyork * 1
    newyork_pred.append(pet_newyork)

    newyork_pred

    set_figsize()
plt.plot(shanghai_pred, 'd-', linewidth=2, color=default_colors[0], label='
                                     Shanghai')
plt.plot(shenzhen_pred, 'd-', linewidth=2, color=default_colors[1], label='
                                     Shenzhen')
plt.plot(newyork_pred, 'd-', linewidth=2, color=default_colors[2], label='
                                     New York')
plt.xlabel('Year')
plt.ylabel('Number of Pets')
plt.legend()
plt.yscale('log')
plt.show()

set_figsize()
plt.plot(shanghai_pred, 'd-', linewidth=2, color=default_colors[0], label='
                                     Shanghai')
# plt.plot(newyork_pred, 'd-', linewidth=2, color=default_colors[2], label
                                     ='New York')
plt.xlabel('Year')
plt.ylabel('Number of Pets')
plt.legend()
plt.yscale('log')
plt.savefig('Shanghai.pdf', bbox_inches='tight')
# plt.savefig('name.svg', bbox_inches='tight')
plt.show()

plt.plot(shenzhen_pred, 'd-', linewidth=2, color=default_colors[1], label='
                                     Shenzhen')
# plt.plot(newyork_pred, 'd-', linewidth=2, color=default_colors[2], label
                                     ='New York')
plt.xlabel('Year')
plt.ylabel('Number of Pets')
plt.legend()
plt.yscale('log')
```

```
plt.savefig('Shenzhen.pdf', bbox_inches='tight')
plt.show()

set_figsize()
plt.plot(newyork_pred, 'd-', linewidth=2, color=default_colors[2], label='
                                        New York')
# plt.plot(newyork_pred, 'd-', linewidth=2, color=default_colors[2], label
                                        ='New York')
plt.xlabel('Year')
plt.ylabel('Number of Pets')
plt.legend()
plt.yscale('log')
plt.savefig('New York.pdf', bbox_inches='tight')
plt.show()
```