Summary

The problem of homeless pets and worsening feral pets living conditions have caught the attentions of the IMMC-A. Henceforth we have been tasked to determine if a household is suitable for adopting a cat and other different species.

In task 1 (the warm up), we first researched on the factors that we thought would affect the evaluation. Then, we determined the deciding factors, those that had a big impact on the result; Namely **Purchasing power**, **area of household** etc. For the next step, we quantified all the elements for efficient and straightforward investigation. However, we believed that simply quantifying the factors would not give us an accurate representation and score for an accurate index; Thence, we decided to normalize the factors such that it gives a fair index. Finally, we made use of the **Analytic Hierarchy** Process to generate weight coefficients and have completed the first version of the model.

For task 1a, we selected Hong Kong as the region for the households for model validation, and chose 6 households with varying income level, house area, and environment. Then, using the above model, we evaluated their readiness to receive a cat.

Considering task 1b, we have chosen **UK**, **USA** and **Hong Kong** as the regions good for cats; Not only do they have diverse environments, but they also have sufficient data for analysis, we believe that these characteristics will improve the accuracy of our model. After selecting the countries, we assigned values for each factor with **Probability Density Functions** for all 3 countries. Finally, we constructed normal distributions for better analysis.

Regarding task 2, we were tasked to generalize our model and allow it to consider other species. Similar to task one, we have researched and chosen 4 distinct pets: **Dog**, **Tegu**, **Hamster**, and **Parrot**. These pets were not only popular, but had varying lifestyles and different handling requirements, we believed that this would allow us to make our model more diverse.

After that, we have **re-quantified** a selection of individual factors to match our new species, this allows our model to give out more accurate results tailored for the new species. For the next step, similar to re-quantifying the factors, we have also decided to remake our weighting matrices.

After initializing the new generalized model, we have evaluated the same 6 households in task 1a on their readiness to get the 4 species on the above as pets respectively. Some households are not ready to receive a dog or a tegu, but all households are suitable for a parrot and a hamster.

Finally for task 2b, we created **radar charts** for each species of pet, with 7 vertexes each representing one factor; We calculated the overlapping area of 2 pets and obtained the compatibility score for the pair. After that

Considering task 3, we were tasked to project future pet ownership and retention. To project the future pet demographics, we have decided to investigate on the change of each factor over time. After that, we simply substituted in the values of 5, 10 and 15 years and evaluated the percentage changes. Finally we multiplied the results into the 10000 set of generalized data in task 1b and calculated the change in the score; We believe this would give us an accurate prediction.

Dear Decision Makers of the IMMC-A,

Re: Recommendation on matching pets and prospective owner households

This is team 2024020 writing to inform you about our model on determining how ready a household is to adopt a pet, thank you for your confidence in our team, and we are determined to produce a detailed model on how we can solve this issue.

After thorough analysis of affecting factors of the household and the cats, our team has discovered a model that can accurately evaluate the readiness of a household for cats and other species of pets, based on the few inputs needed.

For the model, we have considered regional factors like the hygiene and the community support offered for pets within the region. Additionally, we have also considered factors for individual pets like the desired climate and the noise level they are comfortable with. By combining these 2 aspects of factors, we are confident the model will be able to generate an accurate appropriacy score for the household to own a specific pet.

Regarding the solution, our model is already capable to evaluate the readiness for a household to adopt a pet; Unfortunately, we do not have the expertise to provide a thorough evaluation regarding if a household is really suitable for a specific household. Henceforth, we sincerely hope the professionals at IMMC-A with adequate knowledge can aid our model towards perfection by retrieving actual data and adjusting the weights of our model via the pairwise comparison matrix.

Additionally, the threshold for determining if a household is suitable is also not perfectly accurate, we hope professionals will put our model in use to increase the accuracy of our model. The threshold can be changed depending on if there are too many or too little households that meet the requirement. By the process of trial and error, the model can be further corrected.

Last but not least, combining on our model's evaluation and detailed background check, more households can prepare themselves for adopting a pet with the lowest cost, which would definitely improve the situation regrading homeless pets or pets with horrible living conditions.

To encapsulate, our model is capable of generating scores and evaluation on paper; However, words are unfounded, and a combination of actual deployment and our mathematical model is required to generate an accurate result.

Thank you for your attention to this matter.

Yours faithfully,
Team#2024020

Contents

1 A Feline Warm up

1.1 Question Analysis and Restatement

We are asked to first build a quantitative model to compute a household's readiness for cat ownership. The model, being used by pet stores and animal shelters, will intake no more than 10 factors to be more user-friendly.

We restate the problem, and outline our approach as follows:

1. Find out all factors that affect house readiness for pets
2. Choose at most 10 factors that are most appropriate for our model
3. Quantify each factor and transform it into a score that can be normalized for summing up. We call it a *factor score.*
4. Generate the final **readiness score** by determining how important each factor is and sum up all the scores with appropriate weighting.

1.2 Generating factor scores

1.2.1 Preliminary Research

There is an array of factors that determine whether or not a household is suitable for keeping cats as friends. After scavenging and exhausting all the different factors, we have the following results:

1. **Community support**: Outdoor activity space, veterinary care, and pet stores nearby are important to cats. [1]
2. **Purchasing Power**: Lack of purchasing power may cause a lack of cat food, medication, and other products, which may worsen the living conditions of the cat and the financial stress of the owner. [2] [3]
3. **Hygiene**: Illness and infections from the environment may lead to discomfort and fatigue in the cat. [4] [5]
4. **Time In House**: The time when the owner stays home affects the bonding, stress levels, and the mental health of the owner and the cat. [6] [7]
5. **Allergy**: Allergic reactions, discomfort often causes mutual disgust and between the owner and the cat. [8] [9]
6. **Noise Level**: Loud noises may trigger fear and anxiety in cats, which can lead to various health issues. [12] Moreover, irreversible damage to a cat's hearing abilities is also possible. [13]
7. **Climate**: The climate situation of the household can also affect the health of cats, and temperature is considered to be the most important factor in deciding whether the climate is suitable for cats. [20] In order to keep the model's user friendliness, we decided to only focus on temperature as the only climate factor. Cats are susceptible to heatstroke and dehydration in hot weather. [15]

8. **Topology of House**[1]: There may be safety hazards or dangerous elements in the house which may cause damage to the cat. [16] Moreover, insufficient size of the house may also lead to stress and anxiety. [17]

9. **Compatibility with animals:** Some house plants are toxic to cats. Moreover, cats may see other pets as prey. [36]

There were some other factors, such as the demographic of the household and the emotional status of the owner. However, they are out of our consideration since they may have little to no effect on the result. Besides, emotion cannot be easily modelled and to collect data on.

Note that these factors are indeed exhaustive for all pets and we will keep referring to the list at a later stage.

1.2.2 Selecting the factors

In the following, we select the factors useful for our preliminary model at the beginning of task 1. We can see that some factors largely differ from others by nature. For example, the factor of "allergy" is a simple yes or no input while the area of the house is a number. We call "allergy" a *boolean input* and we need not quantify it.
Therefore, we will need to quantify community support, purchasing power, hygiene, time in the house, noise level, climate, and area of the house.

1.2.3 Quantifying the factors

It is obviously important for us to quantify the factors for further analysis, henceforth we will introduce our methods of achieving this goal. In the following, we use a multitude of methods to generate a factor score, $S_{\text{factor}} \in [-1, 1]$ which acts as a normalised score for each factor. A score of 1 means that the household is ideal for that particular factor. A score of 0 means that it is just acceptable. A negative score means that the factor is so bad that it would negatively impact the general score, meaning it requires high compensation from other factor scores to bring it above the threshold of a suitable household.

Quantifying Community Support The support by the community and neighborhood is easily the most important factor to consider, as the community is what provides the majority of resources and services. To measure how much community support each pet can receive, we decided to consider the "pet to vet" ratio of the region the household is situated in.

$$R(X) = \frac{\text{Number of cats in region X}}{\text{Number of vets in region X}} \tag{1}$$

Note that the vet to pet ratio is a predetermined constant, only dependent on the region situated. So, **the input for this factor is the name of the city**.

By seeing how much veterinary care a pet can receive in a community, we can effectively measure community support. Nevertheless, this does assume something.

[1] we define a *house* to be the living area of a household

Assumption 1

The population density of cats and vet clinics/vets are uniform within the input region. This is so that we can gather sufficent data to approximate the pet to vet ratio of any place in the world. **In an ideal situation, the customer flow of the nearest clinics should be measured.**

To compute the score, we first found out that a vet sees about 30 pets a day[52], and a pet sees a vet about 3 times a year [53], and a vet usually have about 250 days of work in a year [54]. This comes out to a pet to vet ratio of $R = 2500$. So, for this standard vet to pet ratio, we take its score to be 0. The optimal pet to vet ratio is 0, meaning that a vet doesn't see any other pets besides the cat of the household. So, we need a score function to satsify:

1. $S \approx 1$ when $R = 0$
2. $S = 0$ when $R = 2500$
3. $S \to -1$ when $R \to \infty$
4. S should change most sharply when R is closer to 2500, as to effectively differentiate the majority of the households.

So, we decided to use a logistics function to model the score function. After some tweaking with the parameters, we found the below function to be most appropriate.

$$S_{\text{Community Support}} = \frac{2}{1 + \exp\left(0.002(R - 2500)\right)} - 1 \tag{2}$$

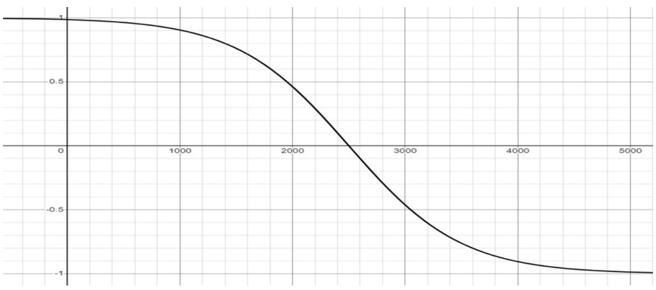

Figure 1: Graph of $S_{\text{Community Support}}$ against pet to vet ratio R

However, we have made an assumption in the data collection process. The number of pet visits (and other data collected later) are dependent with respect to the age of cats.

Assumption 2

We assume **all cats in question are adult cats**, as to ensure the consistency of data collected.

Quantifying Purchasing Power Prospective owners need to afford various things such as food, medical care, toys, etc. Not only can cats worsen the financial burden for low-income households, but also not being able to afford these pet necessities can be detrimental to the cat's physical and mental health.

So, we need to measure the **ability to provide for a cat**. Although the direct way of using the income is trivial, it cannot capture the specificity of cats, as the cost of cats can differ from other pets. After knowing that the cost of raising a cat is 1800 USD per annum in USA [18], and neglecting the marginal one-off cost of getting a cat in the first place. We see that the score for this factor can be expressed as $\frac{1800}{\text{Annual salary (USD)}}$. However, this model would not account for the different commodity prices worldwide. It probably costs less than 1800 USD to have the same necessities in other countries. Therefore, we modify the above model to account for the commodity prices by considering the "cost of living index" of a city. We can get the cost of raising a cat by multiplying 1800 to the ratio $\frac{C_X}{C_{USA}}$, where C_X is the cost of living index in the household country/city X and C_{USA} is the index for USA, which is 72.9. Therefore, we have

$$I = \text{Income ratio} = \frac{1800C_X}{72.9 \times \text{Annual salary (USD)}}. \tag{3}$$

The input is the city name and the annual salary. To transform the income ratio into the factor score, we have to consider the fact that 30% of the income should be spent on non-necessities [55]. So, $S = 0$ when $I = 0.3$. Moreover, once the income ratio exceeds 50%, it starts consuming the cost of necessities[55]. Therefore, $S = -1$ when $I = 0.5$. We then measure the score linearly with I using these 2 points. However, $S > 1$ when $I < 0.1$. So, we have to bound S at 1 when $I < 0.1$, which is still reasonable. If your pet costs less than 10% of your salary, then of course it would be worth it. So, we have

$$S_{\text{Purchasing Power}} = \begin{cases} 1 & \text{if } I \in [0, 0.1] \\ -\frac{1}{0.2}(I - 0.3) & \text{if } I \in [0.1, 0.5] \\ -1 & \text{if } I > 0.5 \end{cases} \tag{4}$$

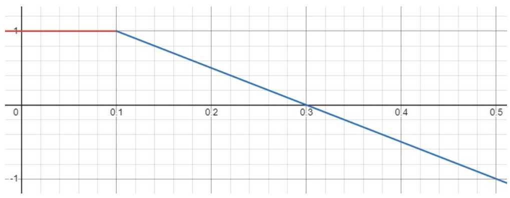

Figure 2: Graph of $S_{\text{Purchasing power}}$ against income ratio I

Quantifying Hygiene The hygiene of the environment is obviously essential for the **optimal living conditions** for both the owner and the cat, as it could be detrimental for the cat and the owner's health. In theory, the hygiene of the household should be measured. However, this is not user friendly, and it is impossible for us to gather data on this aspect. Therefore, we will consider the hygiene levels of the community to determine the score. **The input for this factor is again the city/country name**.

> **Assumption 3**
>
> The hygiene level of the household is similar to that of the input region the household is situated in.

To measure the hygiene level of the input region, we have found 3 indexes commonly known for measuring health and hygiene of a specific country, region; Namely the Global Health Security Index (GHS Index), Environmental Performance Index (EPI) and the Percentage of People Using Basic Sanitary Devices (WSH_SANITATION_BASIC). Then, we took the average of the 3 scores of 173 countries (The number of countries present in all 3 indexes), and normalized them to form the **hygiene index**.

We then need to transform this index into a score. Note that even in the most unhygienic countries, cats (and in fact pets in general) are not significantly affected by the outdoor hygiene. Therefore, any outdoor hygiene can be accepted. In other words, we need not create a negative score. For this factor, we need to find a way to transform the index into a non negative score.

While max-min normalisation is a trivial approach, it was rejected since it is not very accurate with outliers. Here, we first use the z-score normalisation method.
Take the mean of the scores as μ and the hygiene index of the n^{th} highest ranked country as $s(n)$ for the following normalization process.

$$\mu = \frac{1}{173}\sum_{n=1}^{173} s(n) = 59.28921002 \tag{5}$$

$$\sigma = \sqrt{\frac{1}{173}\sum_{n=1}^{173}(s(n)-\mu)^2} \tag{6}$$

Then, we replaced the score of each country according to the following equation, let's call $z(n)$ the **standard hygiene score** of the n^{th} highest ranked country.

$$z(n) = \frac{s(n)-\mu}{\sigma} \tag{7}$$

Finally, we conclude our normalisation process by transforming the standard score to a score between 0 and 1. We accomplish this by using the normal distribution projected from the definition of the standard hygiene score. By considering the bell curve graph, we can define an area function that considers the fraction of area under the bell curve from $-\infty$ to $z(n)$.

$$S_{\text{Hygiene}}(n) = \int_{-\infty}^{z(n)} \frac{1}{\sigma'\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma'^2}\right) dx \tag{8}$$

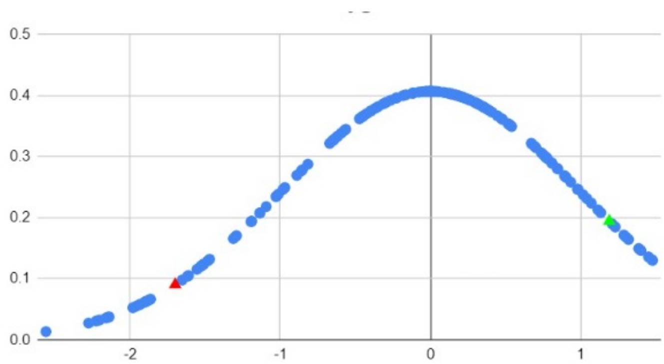

Figure 3: z-score distribution: Red=Cameroon, Green=France

(a) Representation of S_{Hygiene} of Cameroon (b) Representation of S_{Hygiene} of France

Figure 4: Examples of S_{Hygiene} derived from standard hygiene score $z(n)$

Quantifying Time in House It is without a doubt that cats, just like humans, require companionship and accompany; Henceforth it is plain to see the time of which the owner stays in the house is a deciding factor. According to source , owners should spend at least 30 minutes a day with their cats[56], meaning we need $S = 0$ when time spent at home, call it T(hours), satisfy $T = 0.5$. To continue, we have to assume the following:

Assumption 4

- Everyone sleeps 7 hours a day. This is taken from the average sleeping hours worldwide. [57]
- All tenants within a household are working adults, who leave the house for work and return at the same time.
- All tenants only leave their home for work.

This allows us to intake the user-friendly **input of Weekly working hours**[2], and transform it to daily time at home to be:

$$T = 24 - \frac{\text{Weekly Work hours}}{7} - 7 \tag{9}$$

Consequently, we see that any time above 8 hours of home time is optimal by considering the 8:8:8 hour ratio of sleep, work and leisure time. It is also easy to see that 0 hours is the least desirable. In other words, we need $S = -1$ when $T = 0$ and $S = 1$ when $T \geq 8$. For simplicity, we model the score function linearly.

$$S_{\text{Time in house}} = \begin{cases} 2T - 1 & \text{if } T \in [0, 0.5] \\ \frac{2}{15}(T - 0.5) & \text{if } T \in [0.5, 8] \\ 1 & \text{if } T > 8 \end{cases} \tag{10}$$

Note: The time of which the owner stays in the house can also be modified into an input for more accurate results, as we have not reached the 10 input limit.

Quantifying Noise Level Considering the noise level, being too noisy could be disturbing or even damaging for both the cat and the owner. For reference, cats hear sound comfortably at 60 - 80 dB, and sound above 80 dB be uncomfortable to cats, or cause harm to their hearing abilities if the sound is prolonged.[14] At the level of over 120dB, even short and sharp sound can harm a cat's hearing abilities.[13] Note that anything below 80 dB is considered to be acceptable, and there is no such thing as the optimum noise level the household should achieve.
Henceforth, the $S_{\text{Noise Level}}$ should stay at 0 when the noise is less than 80dB. It should decrease to -1 when the noise level reaches 120 dB. To ease the modelling progress, we suppose the score function to decrease linearly. Therefore, we have

$$S_{\text{Noise Level}} = \begin{cases} 0 & \text{if Noise level} < 80\text{dB} \\ \max\{-1, -\frac{1}{40}(\text{Noise level}) + 2\} & \text{if Noise level} \geq 80\text{dB} \end{cases} \tag{11}$$

In theory, the noise level indoors should be measured. but to account for user-friendliness, we assume the following:

[2] Usually the weekly hours is collected instead of daily on monthly in online sources

Assumption 5

The noise level indoors is similar to that outdoors, and the noise level is evenly distributed in the community.

This allows us to **intake city name** only and reference existing databases to find the noise level in later tasks. **In an ideal situation, the noise level should be measured indoors with something like an app.**

Quantifying Climate In the realm of feline needs, similar to humans, climate plays a crucial role in determining the comfort of the cat. Therefore, in this case, this aspect undoubtedly becomes very important. On account of this, we researched about the suitable temperature, highest survival temperature, and the lowest survival temperature of cats, which is 27.5°C, 38°C and -6.67°C [3]respectively. Denote the household's temperature by E°C[21][22]
Given the fact that cats are more cold-resistant than heat-resistant, we need an asymmetrical score function about the line $E = 27.5$. We also need the score to be non-negative since all households maintain a healthy temperature for humans and thus cats. The score function here adds a bonus score for when the climate outdoors is suitable for cats. We decided to use 2 normal distributions, one for temperatures below 27.5°C, and one for above. Let's denote it as $N_1(27.5, \sigma_1^2)$ and $N_2(27.5, \sigma_2^2)$. We believed that the 2σ range should fall between the mean and the lowest/highest survivable temperature respectively. So, $\sigma_1 = \frac{27.5-(-6.67)}{2}$ and $\sigma_2 = \frac{38-27.5}{2}$. Finally, We normalised the distribution such that $S = 1$ when $E = 27.5$. This gives us

$$S_{\text{Climate}} = \begin{cases} f(E) = \frac{h}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{E-\mu}{\sigma}\right)^2\right) & \text{if } E \geq 27.5 \\ g(E) = \frac{k}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{E-\mu}{\sigma}\right)^2\right) & \text{if } E < 27.5 \end{cases} \tag{12}$$

where the factors $h = \dfrac{1}{f(27.5^-)}$ and $k = \dfrac{1}{f(27.5^+)}$ are used for normalisation.

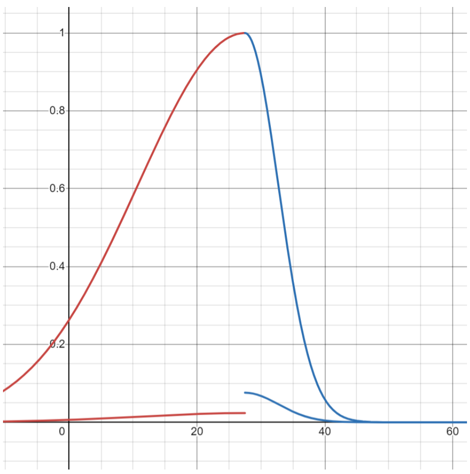

Figure 5: Normal distributions before and after normalisation

Quantifying Household Topology The topology of a house is important for considering cat adoption because it affects the cat's safety, comfort, and overall well-being. Moreover, there could be potential hazards in a household which could endanger the cat. That said, we can only quantify the area of the house .

[3]The 0.01°C comes from Fahrenheit conversion

While using the area of living space directly is intuitive, we also have to consider how cramped the living area is. An easier way of measuring how cramped a living space is looking at how many people are living inside the living area. Let's say a household has n occupants. The average living area used by a single person is 20m^2. So, the extra room left for the cat would be Area of house $- 20n$. We found out that a cat needs at least 4m^2 of extra area to itself[73]. Therefore, the minimum requirement is that the area of house can accommodate 4m^2 more than the space used up by the owners. This corresponds to $S = 0$. Then we design the score function to increase as the the number of 4m^2 chunks increase. We believe that the unacceptable case is when there is no extra space for the cat, corresponding to $S = -1$. Therefore we have,

$$S_{\text{Area}} = \frac{1}{4}\Big([\text{Area of house (m}^2)] - 20n\Big) - 1 \tag{13}$$

bounded by -1 and 1.

1.3 Weighting generation

Now that we have created all seven factor scores, it is time for us to put everything together. Taking the mean directly would be inaccurate since some factors has less on an effect on others. Therefore, we have to decide on some weighting coefficients for each factor score.

There is a multitude of ways we can accomplish this. Here, we decided on the approach of **Analytic Hierarchy Process**. It is a well known algorithm for **Multiple-criteria decision analysis** than can transform "subjective" common sense and non-numerical information into an "objective" numerical weighting, which is less prone to human error. It should be noted that this algorithm is usually used to find out an optimal decision based on layers of criteria. Here, we only have one layer with 7 criteria. However, we will still use a critical part of this method to find out the weightings, with the help of a modelling book we found online. [75]

This crucial part of the AHP is called a **Pairwise Comparison Matrix**. We select 2 distinct factors at a time from the 7 factors and compare them on which is more significant on cats. Let's say we are comparing factor i with factor j. We define a value a_{ij} as follows:

a_{ij}	
1	If i is as important as j
3	If i is slightly more important than j
5	If i is more important than j
7	If i is obviously more important than j
9	If i is absolutely more important than j
2,4,6,8	For the levels in-between

Note that $a_{ij} = a_{ji}^{-1}$. Using this method, we can generate a pairwise comparison matrix $\mathbf{P}_{\text{Cats}} := [a_{ij}]_{7\times7}$. Using some online sources, common sense, and questionnaires, we ran the pairwise comparison algorithm and we developed the following matrix:

$$\mathbf{P}_{\text{Cats}} = \begin{pmatrix} 1 & 1/4 & 6 & 1/5 & 7 & 7 & \boxed{3} \\ 4 & 1 & 7 & 3 & 9 & 9 & 5 \\ 1/6 & 1/7 & 1 & 1/6 & 3 & 3 & 1/2 \\ 5 & 1/3 & 6 & 1 & 7 & 7 & 6 \\ 1/7 & \boxed{1/9} & 1/3 & 1/7 & 1 & 1 & 1/6 \\ 1/7 & 1/9 & 1/3 & 1/7 & 1 & 1 & 1/6 \\ 1/3 & 1/5 & 2 & 1/6 & 6 & 6 & 1 \end{pmatrix} \tag{14}$$

Note that the factors down the row and to the right of the column corresponds to community support, purchasing power, hygiene, time in the house, noise level, climate, and area of the house in sequential order. To give some examples, the 2 boxed entries correspond to 2 pairwise comparisons. One is that purchasing power is absolutely more important than noise, thus giving $a_{52} = \frac{1}{9}$; the other is that community support is slightly more important than area, thus giving $a_{17} = 3$.

To find out the weighting coefficients, we are asked to compute the eigenvector of $\mathbf{P}_{\text{Cats}}$ with the largest real eigenvalue, i.e. the solution $\mathbf{w}_{\text{Cats}}$ of

$$\mathbf{P}_{\text{Cats}}\mathbf{w}_{\text{Cats}} = \lambda_{\text{Cats}}\mathbf{w}_{\text{Cats}} \tag{15}$$

with the largest $\lambda_{\text{Cats}} \in \mathbb{R}$. Using an online calculator, we obtain the required solution to be $\mathbf{w}_{\text{Cats}} = (0.1484, 0.3872, 0.0459, 0.2906, 0.0234, 0.0234, 0.0812)^T$ with $\lambda_{\text{Cats}} \approx 7.749$. These are in fact the weights of the factors respectively. Since the factors scores are rated above 0 if and only if the conditions are acceptable, The threshold for the final score is 0.

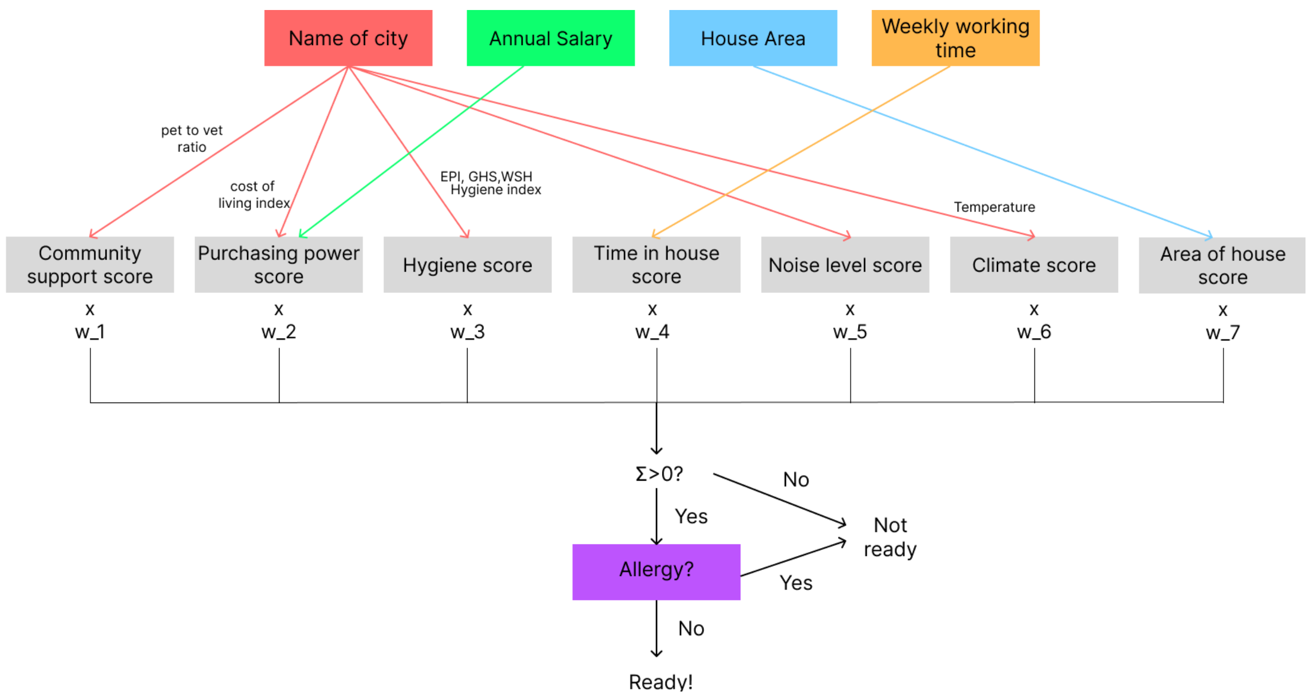

Figure 6: Summary of model

1.3.1 Matrix evaluation and improvement

Since the entries of the matrix are still determined by human input, there is a chance that the ratios are inconsistent. For example, someone may input i is slightly more important than j (3); j is slightly more important than k (3), but i is only slightly more important than k (3). To remedy this inevitable error, Dr. Saaty proposed the following check for consistency:

Define the **consistency index** $CI = \dfrac{\lambda_{\text{Cats}} - n}{n-1}$ where $n = 7$ in this case. Moreover, We define the **randomization index** RI as the consistency index evaluated from the eigenvalues of a large amount of randomized pairwise comparison matrix. For $n = 7$, the corresponding $RI = 1.32$. Finally, we are asked to evaluate the **consistency randomized index** $CR := \frac{CI}{RI}$. The check is that if $CR < 0.1$, the matrix is acceptable. Indeed, for our matrix $\mathbf{P}_{\text{Cats}}$,

$$CR = \frac{7.749 - 7}{7 - 1} \times \frac{1}{1.32} = 0.094 < 0.1 \tag{16}$$

which means that our matrix is acceptable. However, for later tasks, this check was extremely useful when improving out final weighting.

1.4 Model Evaluation

To validate our model‘s ability to capture the diversity of households that could have a cat as a pet, we chose 6 households in **Hong Kong** that highlights different factors. The measured data are as below:

Household	1	2	3	4	5	6
Community Support	410	410	410	410	410	410
Purchasing Power	966	2615	14816	710	3860	7120
Hygiene	0.871	0.871	0.871	0.871	0.871	0.871
House Time a Day	0.876	6.81	9.19	3.95	8.92	13.4
Noise Level	91	63	86	54	77	68
Area of House	16.64	61.8	87.5	42.4	102.6	42.07
Number of People in House	1	3	5	2	3	1
Allergies	no	yes	no	no	no	no

Unfortunately, one member in household 2 is allergic to cats, and therefore his household is not suitable. Hence, we will no longer consider the readiness score of household 2.
Note that some factors in the above are the same among all households. This is because these factors are location dependent, and in this case all households are located in Hong Kong.
After measuring the data in the households, we put the data in our model and calculated the following result:

Household	Result	Readiness
1	-0.165255625	no
3	0.798413956	yes
4	-0.112775751	no
5	0.785495298	yes
6	0.957270913	yes

From the above, households 1 and 4 are not suitable because their readiness scores are negative. Viewing their respective data, we can see that households 1 and 4 have a much lower income, available time, and house area compared to households 3, 5 and 6.

1.4.1 Highlights of Households that qualify

Household 3’s **Purchasing Power** is significantly higher than other households, at 14816 USD per month. This resulted in a high Purchasing Power Index and thereby a high readiness score.
Household 5’s **House Area** is significantly larger than other households, at 102.6 m^2. This resulted in a high House Topology Index and thereby a high readiness score.
Household 6’s **Time in House** is significantly longer than other households, at 13.4 hours per day. This resulted in a high Time in House Index and thereby a high readiness score.

1.4.2 Highlights of Household that do not qualify

Household 1’s **House Area** is significant smaller, at around 16 m^2 (Apartments with such small sizes are common in Hong Kong). Despite that, its income is also relatively low, resulting in a low purchasing power index and house area index, leading to a negative readiness score.
Household 4 ‘s **Income** is significant lower than other households, at around 710 USD per month. This resulted in a negative Purchasing Power Index, and therefore is not suitable for getting a cat.

1.5 A broader Evaluation

In task 1b, we are asked to determine the current number of households that are prepared to own a cat in three countries/regions of our choosing. We have chosen the following countries/regions: **Hong Kong**, **USA**, and **UK**.

1.5.1 Data Distribution of factors

In the chosen 7 quantified factors, only 3 are non-regional dependent, meaning that these factors are different among a region. These factors are: **Purchasing Power**, **Time in house**, and **Household Topology**. According to the assumptions mentioned above, the other 4 factors are assumed to be constant among the same country/region.

In the beginning, we thought of checking the score of the minimum percentile of the population to determine the size of suitable population. However, this assumes that for each household, their ranking of all of the factor are the same which is completely wrong.

Therefore, to **simulate** the number of households that are prepared to own a cat, we decided to take the following approach:

1. Research and get data on the **distribution** of the 3 non-regional factors of Hong Kong, USA and UK respectively.

2. Project the data with the best-fit **probability density function** (PDF) (normal distribution or a gamma distribution function), using existing curve-fitting software such as `matlab histfit`

3. Randomly select 10000 samples based on the probability density function and substitute the samples to check the suitable population size. In order to randomly select a data z according to the distribution, we first generate a random number in the range $(0, 1)$ to represent a random area under the PDF where $P(Z < z)$. Then we find the inverse of the PDF and the required random data can be obtained by $z = F^{-1}(\text{rand}(0, 1))$

4. Using the data generated in PDF and regional factors, evaluate the readiness score of each sample, and calculate the ratio of suitable households to all households.

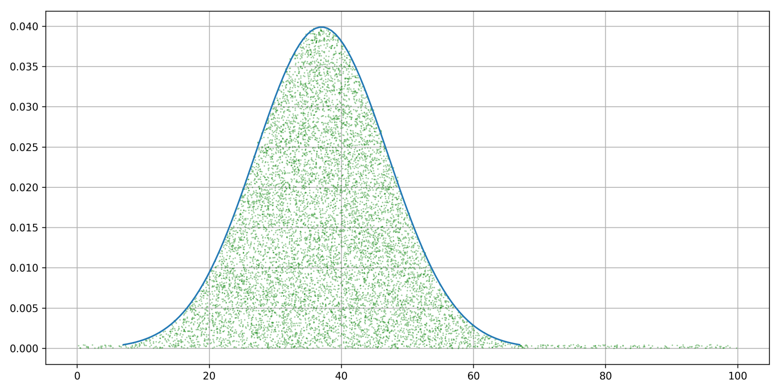

Figure 7: Probability density and 10000 samples of USA working hours, each sample marked as a green dot

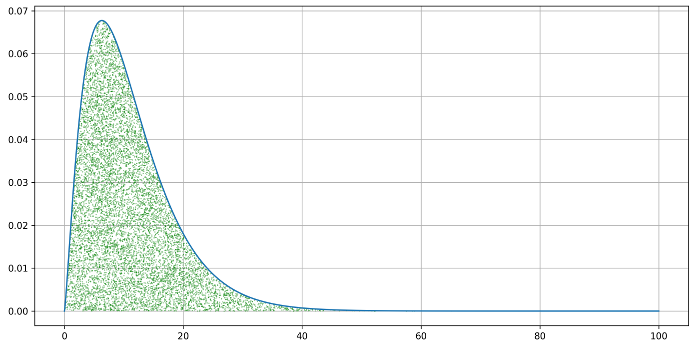

Figure 8: Probability density and 10000 samples of UK income, each sample marked as a green dot

Normal	Mean	SD
UK Area	80.91408591	13.90037377
UK Time	35.70522388	10.99598599
HK Time	41.94174	25.7638
HK Income	27828.68	15947.92
US Area	2250.263992	844.8247088

Gamma	α	β
US Income	1.242360581	8.2485553
UK Income	2.354943918	4.8598581
HK Area	6.935983427	2.3241705
US Area	34.42321446	143.97866

After simulating the data of all 10000 households in a large database we used the model we derived to count the number of prepared households. Fin

2 Model for Other Pets

2.1 Introduction

With one comes more, most pet owners won't stop on one pet, and often finds more companions for the furry baby.
In this section, we generalized our model from the warm up in order for it to be able to calculate the preparedness for other pets or even multiple different species living under the same roof.

2.2 Research on popular pets

Aside from cats, there are also many other popular pets on the market. However, only 4 additional pets are needed for this task; Henceforth, we have decided to shortlist a multitude of popular pets for finalising our decision.

2.2.1 The shortlist of pets

Here are our chosen pets which are all popular choices for people worldwide:

1. Dogs: The first choice that came to our minds, being a man's best friend, there is no question for this selection.
2. Fishes: Another popular choice, however its simplistic lifestyle makes us rethink if a math model is really needed.
3. Parrots: An exotic species favoured by many senior pet owners, its difficulty of maintenance creates huge obstacles.

4. Hamsters: A popular choice for many newbie pet owners, this may seem like a trivial exclusion. However, few know about the difficulty to handle them; Combined with how easy they can get ill, more consideration is needed.

5. Rabbits: They may seem cute and easy to handle; However few know about how easily they can get injured if badly handled.

6. Tegus: A relatively rare choice, however it is the most popular lizard to own with many enthusiast being deeply interested in them.

Note: we have decided on specific species for lizards (Tegus) and birds (Parrots) as the spectrum of species is diverse, with too much data to consider.
We have also found a multitude of other pets, like ferrets and guinea pigs, but their popularity is relatively low compared to the most popular ones; Henceforth we have decided not to further investigate on them.

2.2.2 The selection of pets

We have listed out the top most popular pets to own, now we will exclude a few. To begin with, fishes are an easy exclusion, their simple lifestyle and little affecting factors makes us believe that an entire math model is not needed for evaluating their suitability.
The second pet we have decided to exclude are rabbits. Not only are rabbits the second least popular choices amongst them, there are only a small number of factors that may affect them. As an example, only grooming and attention is needed, henceforth we have decided that rabbits provide little value for our model. In conclusion, we have chosen dogs, parrots, hamsters and reptiles as our choice.

2.2.3 Re-quantifying aspects for the chosen pets

We have considered methods to quantify aspects for cats in the warm up, such as purchasing power and hygiene level of the environment. However, it is crystal clear that most other pets possess different characteristics; Such as dogs, which may require more time with the owner because of their outgoing personality and social lifestyle, and hamsters, which may require less purchasing power but needs more social support because they can easily get ill.
Henceforth, we will now introduce the different methods we have used to re-quantify the aspects for different pets. Moreover, for all the items below, Y represents the species of pet.
Additionally, aspects such as topology of house and hygiene do not need to be re-quantified, as they are regional factors or are mostly independent to the pets.

A special case: Lizards Lizards are very sensitive to their environment and equipment, however due to these restrictions, we have further investigated on the lizards' living environment and found out they live in isolated facilities. [37] Henceforth we have decided that only purchasing power is meaningful to re-quantify when considering lizards.

Quantifying Purchasing Power Similar to that of cats, we will be considering the cost per annum (excluding the price of adopting the animal) and the cost of living index of the region. Finally, $S_{\text{purchasing power}}(X,Y)$ would be the index we are looking for and P_Y would be the cost per annum of handling pet Y. Combined with the variables provided previously, we get the following 2 formulas:

$$I(X,Y) = \text{Income Ratio} = \frac{P_Y C_X}{72.9 \times \text{Annual salary (USD)}}.$$

$$S_{\text{Purchasing Power}}(X,Y) = \begin{cases} 1 & \text{if } I(X,Y) \in [0, 0.1] \\ -\frac{1}{0.2}(I(X,Y) - 0.3) & \text{if } I(X,Y) \in [0.1, 0.5] \end{cases} \tag{17}$$

The following are the quantifications of the different species:

Pet	Cost per annum	$S_{\text{Purchasing Power}}(X,Y)$
Dogs	2082.5 [39]	$= \begin{cases} 1 & \text{if } I(X,Dog) \in [0,0.1] \\ -\frac{1}{0.2}(I(X,Dog)-0.3) & \text{if } I(X,Dog) \in [0.1,0.5] \end{cases}$
Hamsters	75 [40]	$= \begin{cases} 1 & \text{if } I(X,Hamster) \in [0,0.1] \\ -\frac{1}{0.2}(I(X,Hamster)-0.3) & \text{if } I(X,Hamster) \in [0.1,0.5] \end{cases}$
Parrots	1475 [41]	$= \begin{cases} 1 & \text{if } I(X,Parrot) \in [0,0.1] \\ -\frac{1}{0.2}(I(X,Parrot)-0.3) & \text{if } I(X,Parrot) \in [0.1,0.5] \end{cases}$
Tegus	545 [42]	$= \begin{cases} 1 & \text{if } I(X,Tegu) \in [0,0.1] \\ -\frac{1}{0.2}(I(X,Tegu)-0.3) & \text{if } I(X,Tegu) \in [0.1,0.5] \end{cases}$

Table 1: Purchasing Power quantified for the Different Species

Quantifying Noise Level This is also another simple aspect to quantify, no new variables are added. (Note: lizards live in an isolated environment so they do not need a new quantification.)
Quantification of noise levels:

Pet	Harming	Dangerous	Result
Dog [46] [47]	85 dB	140 dB	$n_{\text{dogs}} = \begin{cases} 1 & \text{if } x < 85 \\ -\frac{1}{55}x + \frac{39}{11} & \text{if } x \geq 85 \end{cases}$
Hamsters [50] [51]	85 dB	125 dB	$n_{\text{hamsters}} = \begin{cases} 1 & \text{if } x < 85 \\ -\frac{1}{40}x + \frac{33}{8} & \text{if } x \geq 85 \end{cases}$
Parrots [48] [49]	95 dB	104 dB	$n_{\text{parrots}} = \begin{cases} 1 & \text{if } x < 85 \\ -\frac{1}{9}x + \frac{113}{9} & \text{if } 104 \geq x \geq 85 \\ 0 & \text{if } x > 104 \end{cases}$

Note: The noise index of parrots will never fall to the negatives as they can regenerate their hearing abilities. [48]

Quantifying Community Support Similar to noise levels, only minor factors need to be changed.
Re-quantification of Community Support:

$$R(X,Y) = \frac{\text{Number of pet } Y \text{ in region X}}{\text{Number of vets in region X}} \tag{18}$$

Then, akin to the warm up, we used the logistics function to model the score function.

$$S_{\text{Community Support}}(X,Y) = \frac{2}{1+\exp\left(0.002(R(X,Y)-2500)\right)} - 1 \tag{19}$$

Climate Similar to the warm up, only minor changes need to be made to suit the ideal temperature for the pets.

Pet	Suitable	Highest survivable	Lowest survivable	σ_1	σ_2
Dog [65] [66] [67]	24.72	40	-7	$\frac{24.72-(-7)}{2}$	$\frac{40-24.72}{2}$
Parrot [68] [69] [70]	22.5	40	4	$\frac{22.5-4}{2}$	$\frac{40-22.5}{2}$
Hamsters [71] [72]	21.1	26.7	15.6	$\frac{21.1-15.6}{2}$	$\frac{26.7-21.1}{2}$
Tegu [43] [44] [45]	40.5	57	15	$\frac{40.5-15}{2}$	$\frac{57-40.5}{2}$

Finally, we just need to insert the corresponding values into S_{Climate} and a new quantification would be created for each new pet.

2.2.4 Change in Weighting

For different kinds of pet, the importance of different factors varies. Therefore, the weightings in our model need to be changed accordingly. The new pairwise comparison matrix is as below. Note that these have been adjusted using online sources, forums, questionnaires, and the consistency checks described in task 1.

$$\mathbf{P}_{\text{Dogs}} = \begin{pmatrix} 1 & 1/3 & 6 & 4 & 5 & 7 & 3 \\ 3 & 1 & 7 & 5 & 6 & 9 & 4 \\ 1/6 & 1/7 & 1 & 1/4 & 1/2 & 5 & 1/5 \\ 1/4 & 1/5 & 4 & 1 & 4 & 3 & 1 \\ 1/5 & 1/6 & 2 & 1/4 & 1 & 3 & 1/7 \\ 1/7 & 1/9 & 1/5 & 1/3 & 1/3 & 1 & 1/8 \\ 1/3 & 1/4 & 1/5 & 1 & 7 & 8 & 1 \end{pmatrix} \mathbf{w}_{\text{Dogs}} = \begin{pmatrix} 0.248 \\ 0.398 \\ 0.045 \\ 0.109 \\ 0.049 \\ 0.024 \\ 0.127 \end{pmatrix}$$

$$\mathbf{P}_{\text{parrots}} = \begin{pmatrix} 1 & 1/6 & 1/9 & 1/8 & 1/3 & 1/6 & 1/5 \\ 6 & 1 & 1/5 & 1/4 & 5 & 1 & 3 \\ 9 & 5 & 1 & 2 & 7 & 5 & 6 \\ 8 & 4 & 1/2 & 1 & 6 & 4 & 5 \\ 3 & 1/5 & 1/7 & 1/6 & 1 & 1/5 & 1/2 \\ 6 & 1 & 1/5 & 1/4 & 5 & 1 & 3 \\ 1/5 & 1/3 & 1/6 & 1/5 & 2 & 1/3 & 1 \end{pmatrix} \mathbf{w}_{\text{parrots}} = \begin{pmatrix} 0.022 \\ 0.115 \\ 0.389 \\ 0.278 \\ 0.037 \\ 0.115 \\ 0.044 \end{pmatrix}$$

$$\mathbf{P}_{\text{tegus}} = \begin{pmatrix} 1 & 1/8 & 1/9 & 1/3 & 1/5 & 1/7 & 1/6 \\ 8 & 1 & 1/4 & 6 & 4 & 1 & 4 \\ 9 & 4 & 1 & 7 & 6 & 3 & 6 \\ 3 & 1/6 & 1/7 & 1 & 1/4 & 1/6 & 1/6 \\ 5 & 1/4 & 1/6 & 4 & 1 & 1/4 & 1 \\ 7 & 1 & 1/3 & 6 & 4 & 1 & 4 \\ 6 & 1/4 & 1/6 & 6 & 1 & 1/4 & 1 \end{pmatrix} \mathbf{w}_{\text{tegus}} = \begin{pmatrix} 0.021 \\ 0.199 \\ 0.394 \\ 0.035 \\ 0.076 \\ 0.197 \\ 0.078 \end{pmatrix}$$

$$\mathbf{P}_{\text{hamsters}} = \begin{pmatrix} 1 & 1/3 & 1/8 & 1/6 & 1/4 & 1/7 & 1/4 \\ 3 & 1 & 1/7 & 1/5 & 1/2 & 1/6 & 1/2 \\ 8 & 7 & 1 & 4 & 5 & 6 & 5 \\ 6 & 5 & 1/4 & 1 & 3 & 1/3 & 3 \\ 4 & 2 & 1/5 & 1/3 & 1 & 1/4 & 1 \\ 7 & 6 & 1/6 & 3 & 4 & 1 & 4 \\ 4 & 2 & 1/5 & 1/3 & 1 & 1/4 & 1 \end{pmatrix} \quad \mathbf{w}_{\text{hamsters}} = \begin{pmatrix} 0.024 \\ 0.040 \\ 0.451 \\ 0.139 \\ 0.065 \\ 0.216 \\ 0.065 \end{pmatrix}$$

2.3 Model Evaluation

We will review the pet preparedness of the same 6 households we used in task 1.
Here is our result:

Species	Dog	
Household	Readiness Score	Readiness
1	-0.186442204	no
2	0.54259255	yes
3	0.670984634	yes
4	-0.366865636	no
5	0.886068002	yes
6	0.923963009	yes

Species	Tegus	
Household	Readiness Score	Readiness
1	-0.480697202	no
2	0.315079557	yes
3	0.595811148	yes
4	-0.742825966	no
5	0.567212622	yes
6	0.689869224	yes

Species	Parrot	
Household	Readiness Score	Readiness
1	0.400498846	yes
2	0.763807247	yes
3	0.802154623	yes
4	0.484901592	yes
5	0.890752124	yes
6	0.890752124	yes

Species	Hamster	
Household	Readiness Score	Readiness
1	0.470396028	yes
2	0.600187695	yes
3	0.59453509	yes
4	0.556868902	yes
5	0.723502048	yes
6	0.723502048	yes

Dogs and Tegus require a huge amount of capital and space, therefore households 1 and 4 are not ready to get a dog or a tegu as a pet. However, all households are ready to get a parrot or a hamster as a pet, due to their relatively low capital, spatial and environmental requirements.
It is also worth mentioning that the readiness score represents a household's readiness for getting a specific pet. For example, household 2 is less ready to get a tegu compared to households 3, 5 and 6. If an entity wants to utilize a more conservative version of our model, a threshold can be introduced. For example, the household's readiness score may need to be over 0.5 to be considered ready.

2.4 Interaction of Multiple Pets

In order to determine how suitable are two pets living together, we create a relationship matrix with entries 0 to 1 to measure how compatible the pet weightings are.

2.4.1 Measuring compatibility

In task 2a, we have calculated the weights for each factor of each pet, which can be seen as importance of each factor to each pet. We can generate a radar chart for each pet.

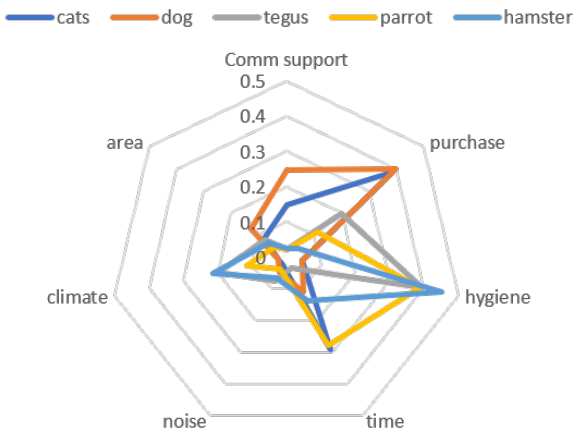

Figure 9: Radar Chart of all 5 pets' weights

The radar chart can describe what kind of household the pets needs most. If two pets share the same needs towards their habitat, it means that they are more suitable together under their habitats, reminiscent of the theory of an ecological niche of an animal. Notice that the overlapping area between the 2 radar charts of 2 pets can represent how similar their needs are. Therefore, we can define the **compatibility score** of two pets is proportional to the overlapping area of the two corresponding radar chart [76].

2.4.2 Deriving the area for overlapping area

To calculate the overlapping area, we divide the octagon into 7 triangular region with a $\frac{360}{7}^\circ$ included angle with 4 points on each side(X_1 , X_2 , Y_1 , Y_2 where X and Y represent different aspects whereas 1 and 2 represent different items) and two segments connecting the points of the same item. This brings us to two cases.

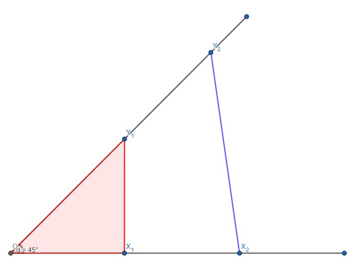

Figure 10: A segment with no intersection of radar charts

Non-intersecting segments Figure 10 shows an example of a slice with no intersection. X_1 in the below equation represents the length from the centre to point X_1 and a similar definition to X_2, Y_1, Y_2. This is a simple case where the overlapping area is the triangle with smaller area.

$$R = \min\left(\frac{X_1 Y_1}{2\sqrt{2}}, \frac{X_2 Y_2}{2\sqrt{2}}\right).$$

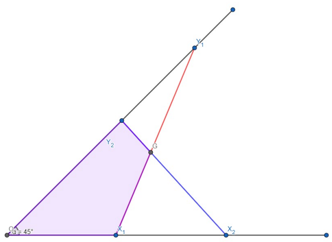

Figure 11: A segment with intersection of radar charts

Intersecting segments Figure 11 shows an example of a slice with no intersection. X_1 in the below equation represents the length from the centre to point X_1 and a similar definition to X_2, Y_1, Y_2. To find this area, we first find the equation of the two segments and find their intersection G(p,q) by solving the simultaneous equations. [76].

$$\begin{cases} y = \frac{Y_1}{Y_1-\sqrt{2}X_1}(x - X_1) \\ y = \frac{Y_2}{Y_2-\sqrt{2}X_2}(x - X_2) \end{cases}$$

The area of the overlapping region can be found by

$$\text{Area} = \frac{Y_1}{2\sqrt{2}}(p - q) + \frac{1}{2}X_2 q$$

Although the above method of finding area is for octagon, by mapping the 2d heptagon into a conical space, the same method can be applied to find the area of the heptagon in the conical space. By multiplying a transformation scaling factor to the area in the conical space, we can obtain the area of the radar overlapping area. The above process will need to be done for all 7 triangles, and the total overlapping area of the 2 heptagonal radar charts will be the sum of all 7 triangles' areas. And as we mentioned, this area will be the magnitude of how complementary the two pets are.

Therefore, the compatibility matrix is,

$$\Omega = \begin{pmatrix} 1 & 0.502 & 0.242 & 0.471 & 0.183 \\ 0.502 & 1 & 0.345 & 0.326 & 0 \\ 0.242 & 0.345 & 1 & 0.373 & 0.276 \\ 0.471 & 0.326 & 0.373 & 1 & 0.492 \\ 0.183 & 0 & 0.276 & 0.492 & 1 \end{pmatrix}$$

If a household is planning to keep a new pet m and the pet existing in their household are $n_1, n_2, \cdots, n_i$ where $n_i \in 1, 2, 3, 4, 5$ and the number represent different pet, .the deduction of the total score due to incompatibility of pets will be

$$S_{comp} = S - 0.5 \sum_{i=1} (1 - \Omega_{m,n_i}) \tag{20}$$

3 Future Pet Ownership

3.1 Effect of time on factors

In order to predict the future trend of pet ownership, we need to understand will different factor change in the future. Therefore, a time function describing the change of each input for our score function is modeled as follow. Note that the distribution of time at home for the general population remains the same therefore it is not considered in this part.
Note, take x as the $(x + 2024)^{th}$ year or x years after 2024.

Factor	Factor over Time Equation
Community Support [59]	$y = \frac{9}{8}(t+24)+9$
Purchasing Power [58]	$y = -437.196796015413t + 40355.78875$
Hygiene [60]	$y = 2\ln(t+10)+65.5$
Noise Level [63]	$y = 20\ln(t+104)+2$
Climate [61]	$y = e^{0.04(t+19)} + 12.8$
Area of House [64]	$y = -3.1\ln(t+9.1)+213$

After substituting the values into the function above, the percentage change of each factor in 5, 10 and 15 years are as follow.

	CS	PP	H	N	C	A
5 years	2.17%	2.86%	1.16%	0.990%	3.17%	-0.964%
10 years	3.15%	5.73%	1.98%	1.94%	7.04%	-1.63%
15 years	4.24%	8.59%	2.61%	2.84%	11.8%	-2.64%

By multiplying these percentage changes into the 10000 set of generated data in task 1b and calculate the resulting change in the score, we are able to predict the size of the population that is suitable for keeping the 5 kinds of pets.

now	cats	dogs	tegus	hamster	parrot
HK	67.20%	56.40%	31.20%	87.20%	74.90%
UK	69.30%	61.30%	34.40%	90.80%	81.40%
US	68.90%	62.10%	41.90%	90.60%	82.10%

5y	cats	dogs	tegus	hamster	parrot
HK	70.56%	60.35%	34.48%	88.94%	76.77%
UK	72.77%	65.59%	38.01%	92.62%	83.44%
US	72.35%	66.45%	46.30%	92.41%	84.15%

10y	cats	dogs	tegus	hamster	parrot
HK	74.09%	64.57%	38.10%	90.72%	78.69%
UK	76.40%	70.18%	42.00%	94.47%	85.52%
US	75.96%	71.10%	51.16%	94.26%	86.26%

15y	cats	dogs	tegus	hamster	parrot
HK	77.79%	69.09%	42.10%	92.54%	80.66%
UK	80.22%	75.10%	46.41%	96.36%	87.66%
US	79.76%	76.08%	56.53%	96.15%	88.41%

4 Conclusion

In conclusion, we have developed a set of metrics that can give a score on whether a household is suitable for raising a cat or other species.

4.1 Model evaluation

Our model used multiple factors that we believed would be able to determine if a household is suitable for evaluating the appropriacy of raising an animal in a household.

4.2 Strength of model

To begin with, our method of normalization and usage of the Analytic Hierarchy Process, we can ensure a fair weight for our indexes to generate an accurate result. Moreover, we applied the power of programming and was able to randomly generate the data of a large number of households; This allowed us to further validate our model with such a large amount of test cases.

4.3 Things to Improve

First off, we have had a few assumptions that would not be realistic; As an example, the assumption of uniform sleep time and uniform distribution of pets and vet clinics are not realistic assumptions.

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AI report 1

LLMs such as ChatGPT were not used, and played no role in the process of creating the mathematical model and this solution paper.

A code

Note that the code below is not generated by AI.
gamma.py:

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.stats import gamma
import random as r
from scipy.stats import norm
import pandas as pd


graph = "nd"
if graph == "gamma":
    a, b = 34.4232144626391, 143.97865997308003
    shape = a
    scale = b
    x = np.linspace(0, 100, 10000)
    y = gamma.pdf(x, shape, scale=scale)
else:
    mean = 37
    std_dev = 10
    x = np.linspace(mean - 3 * std_dev, mean + 3 * std_dev, 100)
    y = norm.pdf(x, mean, std_dev)
```

```
plt.plot(x, y)
plt.grid(True)
plt.show()


new_coords = []

n = 0
while n < 9999:
    coord = (r.random()*100, r.random()*0.08)
    x_index = np.abs(x - coord[0]).argmin()
    if coord[1] <= y[x_index]:
        new_coords.append(coord)
        plt.plot(coord[0], coord[1], 'go', label='Under Curve', markersize=1)
        n += 1
        print("Plotted:", n)
print(new_coords)

plt.plot(x, y)
plt.grid(True)
plt.show()

df = pd.DataFrame(new_coords, columns=['x', 'y'])

df.to_excel('data.xlsx', index=False)

print("Data saved to data.xlsx")
```

radarcal.py:

```
import openpyxl
from sympy import symbols, Eq, solve, sqrt
import math


def cal_area(aspect, a, b):
    sum_area = 0
    for j in range(len(aspect[a])):
        n = j + 1 if j < 6 else 0
        x1 = aspect[a][n]
        y1 = aspect[a][j]
        x2 = aspect[b][n]
        y2 = aspect[b][j]

        if (y2 > y1 and x2 > x1) or (y2 < y1 and x2 < x1) or (y2 == y1 and x2 == x1):  # i
            tmp_area = 1/2*min(x1, x2) * min(y1, y2)*math.sin(math.pi/4)
        else:
            x, y = symbols('x y')

            eq1 = Eq(y1/(y1-sqrt(2)*x1)*(x-x1), y)
```

```
            eq2 = Eq(y2/(y2-sqrt(2)*x2)*(x-x2), y)

            sol = solve((eq1, eq2), (x, y))
            p = sol[x]
            q = sol[y]

            tmp_area = min(y1, y2)/(2*math.sqrt(2)) * (p-q) + 1/2*min(x1, x2)*q
        sum_area += tmp_area
    return sum_area


wb = openpyxl.load_workbook('sus.xlsx')
sheet = wb['sus']

data = []

for row in sheet.iter_rows(values_only=True):
    data.append(row)

# Print the data
for row in data:
    print(row)

animals = ["cats", "dog", "tegus", "parrot", "hamster"]
for i in range(5):
    for j in range(i, 5):
        if i == j:
            pass
        print(animals[i], animals[j], cal_area(data, i, j))
```