2023 International Mathematical Modeling Challenge (IM^{2}C)®

The 9th annual International Mathematical Modeling Challenge (IM^{2}C)® culminated with two Outstanding Teams. Congratulations to these teams and all the teams that participated in the 2023 IM^{2}C. IM^{2}C makes resources available to schools and countries/regions of the top teams to allow them to fund local ceremonies, which they can schedule as their situations permit.

The IM^{2}C continues to be a rewarding experience for students, advisors, schools, and judges. A total of 55 teams, with up to 4 students each, representing 31 countries/regions competed in this year's international round.

The purpose of the IM^{2}C is to promote the teaching of mathematical modeling and applications at all educational levels for all students. It is based on the firm belief that students and teachers need to experience the underlying power of mathematics to help better understand, analyze, and solve real world problems outside of mathematics itself—and to do so in realistic contexts. The Challenge has been established in the spirit of promoting educational change.

For many years there has been an increased recognition of the importance of mathematical modeling from universities, government, and industry. Modeling courses have proliferated in undergraduate and graduate departments of mathematical sciences worldwide. Several university modeling competitions are flourishing. Yet at the school level, even amid signs of the growing recognition of modeling's centrality, there are only a few such competitions with many fewer students participating. One important way to influence secondary school culture, and teaching and learning practices, is to offer a high-level prestigious secondary-school contest that has both national and international recognition. With this in mind, we founded the International Mathematical Modeling Challenge (IM^{2}C) in 2014 and launched the 1st annual Challenge in 2015.

The IM^{2}C is a true team competition held over a number of days, with students able to use any inanimate resources. Real problems require a mix of different kinds of mathematics for their analysis and solution. And, real problems take time and teamwork. The IM^{2}C provides students with a deeper experience of how mathematics can explain our world, and the satisfaction of applying mathematics to a real world problem to develop a model and solution.

Plans for 2024

We invite countries to enter up to two teams, each with up to four students and one teacher/faculty advisor. The contest will begin on February 12 and end on April 29. During that timeframe, teams will choose five (5) consecutive days to work together on the problem. The faculty advisor must then submit the paper and certify that students followed the contest rules.

The International Expert Panel will judge the papers in early June and will announce winners by late June. Papers will be designated as Outstanding, Meritorious, Honorable Mention, and Successful Participant with appropriate plaques and certificates given in the name of students, their advisor, and their schools.

Plans for the 2024 awards are still being finalized. Complete information about IM^{2}C is at www.immchallenge.org

IM^{2}C Funding

Funding for planning and organizational activities is provided by IM^{2}C co-founders and co-sponsors: *Consortium for Mathematics and its Applications* (COMAP), a not-for-profit company dedicated to the improvement of mathematics education, and *NeoUnion ESC Organization* in China Hong Kong (SAR).

The IM^{2}C International Organizing Committee

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COMAP, USA – Chair

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Mogens Allan Niss,
Roskilde University, Denmark

Ross Turner,
Australian Council for Educational Research, Australia

Jie "Jed" Wang,
University of Massachusetts, USA

The 2023 IM^{2}C Problem: Using Land: A Valuable Resource

The Context

Optimal land use planning and the balancing of community values and business profits often require models that include geography, climate, business options, and community needs, and local culture to make important decisions. The community leaders and business planners are trying to decide the '*best use*' of an available 3 square kilometer parcel of land available for development. A satellite view of the property *is provided in Figure 1*. The property's boundaries are defined by five roads:

- Northern boundary: Maiden Lane
- Eastern boundary: Upton Road
- Southern boundary: Maroney Road
- Southwest boundary: Red Creek Road*
- Western boundary: County Line Road#

* Listed as "Red Creek" on the map (Figure 1), but may be identified as Kasson Way or County Road 108 on other maps.

#Listed as "County Line" on the map (Figure 1), but may be identified as County Road 118 on other maps. This roadway also serves as the boundary between Cayuga County, New York (NY) and Wayne County, NY.

A larger version of the maps in Figure 1, as well as several additional maps of the property highlighting different aspects or the property and showing the property from different vantage points, are available to support your modeling and are accessible in the **Appendix**. Information about the current terrain of the parcel is noted in Figure 2. The terrain statistics include elevation, slope, **aspect**, tree and land cover data for the interior of the parcel.

The land is located in a rural area at a latitude of 43°N in a temperate marine climate with all four seasons including a snowy winter and has adequate water and power supplies. The soil is sufficiently rich for crop farming or grazing animals. Syracuse, NY, USA, an urban population center is approximately 50 kms away with adequate roads and transit systems to access the land.

So far in their process, the decision makers have considered the *options* of an outdoor sports complex, cross-country skiing facility (3-month season), a crop farm, a grazing farm/ ranch, a **regenerative farm**, a **solar array,** an **agrivoltaic farm,** and an **agritourist center**. They are willing to consider other options or even divide the property into sections for different uses. They need your help to model the options and construct a decision method that will make 'best use' of the land.

Your Task:

1. Determine a **quantitative** decision **metric** that defines "best" so the decision makers can feel confident in their final use of the land. The metric should consider short- and long-term benefits and costs.

2. Choose at least two of the options listed above and determine the values of those options in your "best" metric. You may need to find data. You will need to decide which factors to focus and why and make assumptions including on values in order to use your metric. Explain and defend your values or use a range of values to better understand the effects and sensitivities of your assumptions.

3. In October 2022, it was announced that Micron Technology, Inc. will build a very large semiconductor fabrication facility (fab) in Clay, NY, USA, a town just north of Syracuse, NY. Soon after the announcement, it was reported by news outlets that "If fully built, the fabs could employ up to 9,000 people making an average of $100,000 each year. They would create some 40,000 other jobs among suppliers, construc- tion firms and other businesses. the new plant will directly support 9,000 jobs and create nearly 40,000 additional jobs."[2] How will the new fab impact your metric? Re-evaluate the options you identified in the previous question using your "best" metric.

You may *also* consider alternative options for using the land (either an additional item from the initial list

***Figure 1**. Left, a satellite image of the parcel of land. Right, location of the parcel of land. (Shaded region.)*[1]

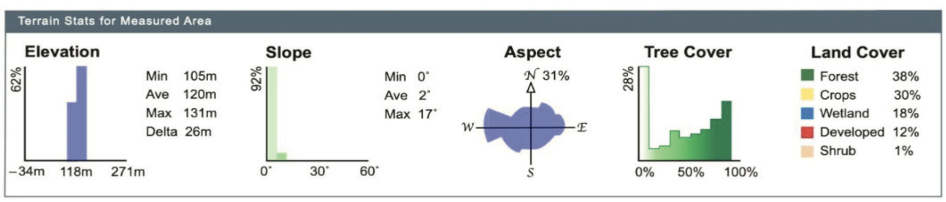


***Figure 2**. Terrain statistics for the parcel of land.*[1]

or any other option not listed above) and evaluate that option using your "best" metric; justify your decision to consider and evaluate another option.

4. Briefly (no more than 1 page) discuss how appropriate your model would be for use in an environment you are familiar with. Make some comments about what might have to change if the land was in a different location or country. That is, consider how generalizable your model is to other locations.

Your PDF submission should consist of:

- One page Summary Sheet.
- One page Letter to the Decision Makers with your recommendation. The audience is the 'The community leaders and business planners' tasked with solving the problem who have asked for your advice. They are already familiar with the problem (and have just read your Summary Sheet so don't restate or repeat this). The purpose of the letter is to provide important information to the decision makers, communicating key details of your recommendation.
- One page Table of Contents.
- Twenty pages (maximum) communicating essential aspects of your solution.
- The following items do not count toward the 23-page limit: Reference List and Appendices.

Note: Your PDF submission must be A4 or letter size, written using no smaller than 12-point font size. For detailed information about IM^{2}C submission guidelines and the general expectations for each portion of your solution please review the **Full Submission Guidelines.**

To see the full problem statement go to: **https://immchallenge.org/Contests/2023/2023_IMMC_Problem.pdf**

The 2022 IM^{2}C International Judges'Commentary

Benjamin Galluzzo

Introduction

A plot of land has many uses, but what's the "best" use? The 2023 IM^{2}C problem challenged teams to justify and define an approach to quantify ideal ways to utilize a plot of land near Syracuse, New York, USA. Teams were provided with information about the three-square kilometer parcel of land and asked to evaluate various land use options with the goal of developing a model that could serve as a decision-making tool for community leaders. The problem pushed teams to develop comprehensive metrics that could effectively balance both short-term and long-term impacts and introduced an added layer of realistic complexity to the problem by requiring teams to use their models to account for the impact of a (soon to be built) nearby semiconductor fabrication facility on the metrics and land use options. This required teams to adapt their models to account for changes in variables like job availability, population density, and even ecological factors.

Identifying and creating a thorough approach to quantify land use options resulted in most teams developing optimization models. Nearly all teams considered maximizing economic growth, while some teams included additional objectives such as minimizing environmental impact. In general, the IM^{2}C judges were impressed by the students' modeling skills, creativity, mathematical knowledge, and writing abilities. The judges appreciated good modeling in terms of explaining and justifying assumptions; explaining the steps of their modeling methods; identifying the strengths, weaknesses, opportunities, and limitations of their models; conducting a sensitivity analysis of parameter values; and presenting their work in a well-organized report.

The problem requirements of this year's IM^{2}C asked teams to:

- Consider and compare at least two land use options for the identified parcel of land.
- Evaluate their models' adaptability to provide accurate results in the event of changes in the immediate area as well as it's capabilities in alternative geographic regions.
- Write a letter to community leaders executive outlining the team's results.

Teams did well in identifying and defining the problem's variables and parameters, researching the elements associated with land usage and building viable models. The judges congratulate the teachers and advisors who developed modeling skills in their students and prepared teams for this year's Challenge.

Problem Solutions

The teams' reports included a summary sheet, a restatement of the problem from their own perspective and in their own words, a discussion of the mathematical modeling processes used (especially, assumptions with justifications, good mathematical notation with defined variables, a mathematical model, the application of the model to the problem requirements, and analysis of the results). Most teams identified

USA Participation

In the USA, we invite all teams that successfully compete in the HiMCM contest and are awarded a designation of Meritorious or above (Meritorious, Finalist, or Outstanding) to compete in the IM^{2}C. From these participants, U.S. Judges select the two top teams to move on and represent the USA in the IM^{2}C international round. To participate in HiMCM in November 2023, visit: **www.comap.org** their model's strengths and weaknesses and wrote a conclusion summarizing their modeling work. Teams also wrote a letter to local decision makers communicating key details of their recommendation. The detailed elements of the problem solution with commentary are included below:

Summary: Ideally papers will start with a short, one-page summary that describes the problem, the modeling methods used and highlights outcomes. This summary is a key part of the IM^{2}C report because it serves as the first opportunity for a team to share their approach, model(s), and results with a reader. It should clearly explain how they tackled the problem and their main takeaways. Judges typically read the summary first to get a basic idea of the paper's methods, findings, and recommendations. However, some teams either gave too much detail about one part of their work or neglected to explain their main findings and advice. The best summaries are both easy to understand and concise; inviting the reader to keep turning the pages to learn more about the model and the results.

Problem Restatement: Teams often restated the problem in their own words by identifying the specific requirements they focus on and the organization of their work. Judges use this part of the report as a preview and overview as to how the team approached the problem and the terminology and notation the paper uses.

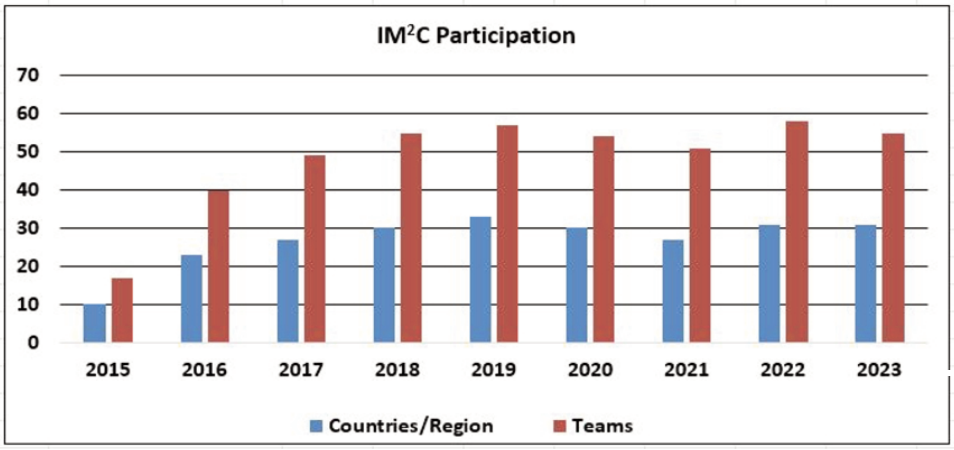
Mathematical Modeling Process: This year's problem required teams to develop a model to determine the "best" way to utilize a plot of land. While teams were given freedom to explore many modeling perspectives, the models they developed had to address the following challenges:

- Define "best" in terms of the problem statement; notably short- and long- term time periods need to be considered.
- Return a value or "score" to compare (at least two) land use options. Teams could explore any land use option for the area but were provided with eight in the problem statement: an outdoor sports complex, cross-country skiing facility (3-month season), a crop farm, a grazing farm/ranch, a regenerative farm, a solar array, an agrivoltaic farm, and an agritourist center.
- Accommodate for changes occurring in the region of interest. In problem three, teams need incorporate the (likely) development of a new factory into their calculations and their land use recommendation.

The definition of "best" was typically framed in terms of the factors that teams chose to incorporate into their model and illustrated by the output.

The 2023 IM^{2}C Outstanding Teams

School, Location	Advisor	Team Members
Ridley College Canada	Gary Pimentel	Alex (Zixuan) Li Bobo (Ziang) LI Michael (Yan) Xiao Jocelyn Wang
Heinrich-Heine-Gymnasium Germany	Barbara Burckhardt	Hatim Abdel Ghaffar Marten Maager Jonas Alexander Daniel Nickel Aaron Oliver Patschula



Numbers of Participating Countries/Regions and Teams 2015-2023

The 2023 IM^{2}C Expert Panel

Benjamin Galluzzo,
Clarkson University, USA – Chair

Maxim Davydov,
Novosibirsk State University, Russia

Ruud Stolwijk,
Cito, The Netherlands

Liqiang Lu,
Fudan University, China

Jill Brown,
Deakin University, Australia

Daniel Long,
Chinese University of Hong Kong, China Hong Kong (SAR)

Dra. Ángeles Domínguez Cuenca,
Tecnológico de Monterrey, Mexico

For example, many teams defined “best” as the land use approach that will make the most money over a certain time period. Teams that considered multiple factors were confronted with the additional challenge of determining (and justifying) the scale of influence each factor has on the model’s output. Teams approached this challenge in a variety of ways; many developing a model from the “scratch” with other teams employing more complex multi-criteria decision-making methods such as Analytic Hierarchy Process (AHP) or Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS). Judges consider all models equally, but regardless of the method a team chooses to employ all aspects of the model must be clearly explained; furthermore, the choice of model needs to be justified. That is, all models and modeling approaches are held to the same standard. Teams that build a model should highlight aspects of the process they use to develop the model as it relates to the problem. While teams that utilize existing models need to provide an explanation of the model (i.e., how it works) as well as their reasoning for choosing that approach (i.e., why it is appropriate to use in this situation).

Application of the model and modifications: All teams built a model and used the resulting metric to compare at least two different options for land use. The teams then chose how (or if) to adjust parameters associated with their initial model to find a solution if the Micron semi-conductor fabrication facility is built in nearby Syracuse. Many teams employed multiple models to obtain their results. One common approach was to build a model to sub-divide the parcel and then identify the “best” use of each sub-parcel via their land utilization metric. In some of these cases, graphics, such as flow charts, were used to illustrate the connection between sub-models and the develop- ment of the team’s model. Several teams created maps of the region that highlighted how their model allocated sub-parcels of land. As required, teams also briefly discussed the generalizability of their models by considering how (or if) their model would still be effective if tasked to rate land use options at different locations.

Sensitivity, strengths, weaknesses, conclusions, and references: Sensitivity analysis is a critical component of mathematical modeling, serving as a diagnostic tool to assess the robustness and reliability of a model’s predictions. By systematically varying the input parameters of a model, sensitivity analysis reveals how changes in these inputs can impact the model’s outputs. This is invaluable for several reasons. First, it helps to identify which parameters have the most significant influence on the results, allowing modelers to prioritize data collection or refinement in these areas. Second, it underscores the uncertain- ties inherent in modeling, ensuring that stakeholders have a comprehensive understanding of the potential variability in predictions. Moreover, in decision-making contexts such as this year’s Challenge problem, sensitivity analysis can be pivotal in assessing the risks (and rewards!) associated with different choices, as it provides a range of possible outcomes based on the variability of input parameters. While a full sensitivity analysis may not be possible due to time constraints, some teams were able to investigate the sensitivity of their models to parameters that they identified as important. for example, a few teams tested the impact of plus/minus percentage changes in costs associated with constructing different land use options. Most teams included a discussion of the strengths and limitations of their models. As statistician George E.P. Box wrote nearly 50 years ago, “All models are wrong, but some are useful.” Under- standing the possible deficiencies in your model enhances the trustworthiness of your model. Similarly, citing all resources used to develop a model increases credibility of the modelers. Teams constantly included a list of resources and references in their submission. While the IM^{2}C doesn’t require a specific reference style; it’s recommended you use a consistent style.

One-page letter to leaders and planners: Teams wrote a letter to community leaders and business planners in which they shared the pertinent details of their model and modeling solutions. Like the summary, good letters were clear and concise. How- ever, a good letter differs from the summary by sharing fewer technical details and instead dedicating more time to sharing results and recommendations. For example, some letters offered multiple recommendations based on choices (i.e., maximum, or minimum allowable subdivisions or land use types) that would ultimately be made by individuals in local or regional leadership positions.

Goals of the Challenge and the roles of the judges

The purpose of the IM^{2}C is to promote the teaching of mathematical modeling and applications at all educational levels for all students. By providing a venue to experience mathematical modeling, the IM^{2}C hopes to inspire student modelers to make appropriate assumptions that lead to viable approaches, use inventive and creative ideas as needed, and apply the mathematics they know in the models they build and implement. By accomplishing these goals during the IM^{2}C, students develop new skills in modeling and refine and practice the skills they already possess. This year’s IM^{2}C teams were able to show their modeling skills by making appropriate choices for their models and successfully implementing their models to “score” provided land use options and determine how the plot of land identified in the problem statement should be developed. Most teams utilized some form of optimization as the basis for their primary model; specific approaches varied greatly. Choosing when and how to use computational tools is often an important decision for modelers. While not required, some teams wrote computer code to expedite calculations associated with their models. Since IM2C does not require inclusion of the computer code in the report, a description of the code, a flowchart, or a simplified pseudocode were good ways to explain their model in the report. Some teams included their code in an appendix, but, as the Challenge's rules state, the judges did not necessarily read the code. More important than the code for the IM^{2}C is the model itself and the steps taken in developing the model and calculating the results.

By reading the papers, the judges evaluated the teams modeling process and determined how well the student teams:

- Created and justified (i.e., through assumptions) their models and parameter values.
- Demonstrated creativity in the different elements of the model.
- Communicated their model to the reader.

The judges had the opportunity to read many excellent submissions that developed innovative algorithms for identifying the "best" way to utilize a parcel of land. The judges commend all the participating teams for dedicating time and effort to truly engage in mathematical modeling this past year.

Some Examples of Good Modeling
Of the 55 papers, 13 were judged Successful, 35 were awarded Honorable Mention, five achieved Meritorious, and two were judged as Outstanding. The strongest teams demonstrated an understanding of the processes and structures involved in the problem and utilized their knowledge to build a viable model. Some of the characteristics found in the best papers, including a few innovative approaches are described below:

- A number of approaches were used to split the land into smaller portions. A nice approach was to use a fixed grid system and then identify a value (based on a team's model) for each sub-parcel.
- In addition to rating sub-parcels and assigning each a land use option, a few teams noticed that their model recommended they build a number of "disjoint" facilities (i.e., non-adjacent sub-parcels designated for the same land use option). Recognizing that this result would likely be unrealistic, the teams adjusted their models to address this situation and avoid stand-alone sub-parcels. One team addressed this issue initially, by applying Sliding Window Analysis from bio statistics to partition the land into contiguous sub regions.
- Teams were provided with six maps of the region for this year's contest. Perhaps in response, a number of teams gave at least a portion of their results in the form of a map; many of which clearly displayed their recommendation. In some cases, maps were used to highlight how and why a model was updated. For example, one of the teams who noticed their algorithm had returned disconnected and isolated sub-regions with similar land use recommendations used maps to share how they adjusted their model to develop a more feasible result.
- A number of teams developed models that considered additional factors other than monetary gains (or losses). Some teams incorporated aspects such as environmental sustainability and tourism. Justifying the inclusion of such factors isn't trivial. One team wanted to consider a number of (primarily) environmental sustainability factors for each portion of land, they used a binary approach to address each of the factors on a parcel-by-parcel basis. Another team based their choices on the three pillars of sustainable development as outlined in the United Nations Sustainable Development Goals; Economic, Social, and Environmental.
- Sensitivity analysis was conducted on parameters such as the cost associated with building different facilities associated with land use options. Using this approach, some teams were able to identify threshold values that might result in a change in land use policy.
- As noted, many teams considered multiple factors when building their models and in doing so needed to identify and justify the impact that each factor has on model's output; this isn't easy. One team that considered both environmental and economic effects used an easy to interpret graphic to share the changes to their model's output associated with a wide range of weight changes for their two key factors.
- There were a number of very good diagrams and charts (not just maps). Graphics are a great way to clearly explain your modeling process to a reader and/or share results in a concise manner.

Advice to Future Teams
As a valuable tool for problem solving and issue analysis, modeling seeks to describe a real-life situation using appropriate mathematics. For the IM^{2}C, teams should organize themselves into a productive team so they can focus their efforts on the requirements of the problem and write a report in a short period of time. Budgeting time is critical so that there is enough time both to solve the problem and to communicate your work

and results. Judges are not looking for the papers that use the most sophisticated mathematics so do not force the use of mathematics, instead use the mathematics that you know and understand. As time allows, refine and enhance the model to increase its precision, or adjust assumptions to find a more broadly appropriate solution. List any sources used during Challenge and document how they were used. Overall, present the development and analysis of the modeling in a manner that a wide audience understands. Conclude the report with a summary of results and recommendations. Some specific comments from the judges are listed below:

- Consider alternative approaches for sharing variables in your paper. A single page (or multiple pages) of variables is difficult to read and makes it hard to remember how the variables have been defined. That is, once you reach the equation that contains the variable in question, it may have been introduced pages earlier. You might try waiting to introduce variables until they are used in your paper.

- An international panel of judges are reading your paper. Because it's likely that you share some different perspectives; you might consider providing context to your modeling choices. In short, explain why your modeling approach matters.

- Make sure to address all parts of the Challenge problem as well as the data. For example, if you choose not to use a portion of the provided data explain the reason for your decision.

- When you choose to use a model found in literature or from an online source you need justify the choice of the model and describe how the model works.

Conclusion

The IM^{2}C judges value the solutions that you share with us, but we are equally interested in understanding how you arrive at your results. That is, they hold both creativity and clarity in high regard. This year's submissions introduced numerous innovative approaches to understanding how mathematical modeling can be used to help us understand complex situations and ultimately make important decisions. The judges, who are experienced modelers and teachers from a wide range of countries, compliment this year's teams on their efforts and the team members for their participation. The judges thank all the schools, teachers and advisors for making it possible for students to participate. The judges wish all the participants success in their future modeling and mathematical endeavors.

For more information about the IM^{2}C, including the complete 2015–2023 results and sample papers, visit

www.immchallenge.org