

INTERNATIONAL MATHEMATICAL MODELING CHALLENGE



CONTEST RESULTS AND PAPER

IM²C promotes the teaching of mathematical modeling and applications at all educational levels for all students. It is based on the firm belief that students and teachers need to experience the power of mathematics to help better understand, analyze, and solve real world problems outside of mathematics itself – and to do so in realistic contexts. The Challenge has been established in the spirit of promoting educational change.

2020 IM²C

The 6th annual International Mathematical Modeling Challenge (IM²C) culminated with two Outstanding Teams. Congratulations to these teams and all the teams that participated in the 2020 IM²C. Due to the closing of venues caused by Covid-19 there was no formal IM²C awards ceremony in 2020. Rather, IM²C made resources available to schools and countries/regions of the Outstanding and Meritorious teams to allow them to fund local ceremonies that they could schedule as their situations permitted.

The IM²C continues to be a rewarding experience for students, advisors, schools, and judges. A total of 54 teams, with up to 4 students each, representing 30 countries/regions competed this year.

The purpose of the IM²C is to promote the teaching of mathematical modeling and applications at all educational levels for all students. It is based on the firm belief that students and teachers need to experience the underlying power of mathematics to help better understand, analyze, and solve real world problems outside of mathematics itself—and to do so in realistic contexts. The Challenge has been established in the spirit of promoting educational change.

For many years there has been an increased recognition of the importance of mathematical modeling from universities, government, and industry. Modeling courses have proliferated in undergraduate and graduate departments of mathematical sciences worldwide. Several university modeling competitions are flourishing. Yet at the school level, even amid signs of the growing recognition of modeling's centrality, there are only a few such competitions with many fewer students participating. One important way to influence secondary school culture, and teaching and learning practices, is to offer a high-level prestigious secondary-school contest that has both national

Plans for 2021

We invite countries to enter up to two teams, each with up to four students and one teacher/faculty advisor. The contest will begin in March and end in May. During that timeframe, teams will choose five (5) consecutive days to work together on the problem. The faculty advisor must then submit the paper and certify that students followed the contest rules.

The International Expert Panel will judge the papers in early June and will announce winners by late June. Papers will be designated as Outstanding, Meritorious, Honorable Mention, and Successful Participant with appropriate plaques and certificates given in the name of students, their advisor, and their schools.

Plans for the 2021 awards are still being finalized. Complete information about IM²C is at www.immchallenge.org

The IM²C International Organizing Committee

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and international recognition. With this in mind, we founded the International Mathematical Modeling Challenge (IM²C) in 2014 and launched the 1st annual Challenge in 2015."

The IM²C is a true team competition held over a number of days, with students able to use any inanimate resources. Real problems require a mix of different kinds of mathematics for their analysis and solution. And, real problems take time and teamwork. The IM²C provides students with a deeper experience of how mathematics can explain our world, and the satisfaction of applying mathematics to a real world problem to develop a model and solution.

The 2020 IM²C Problem: Flash Sale

The 2020 problem statement is printed here for reference. To view the complete problem, go to

http://immchallenge.org/Contests/2020/2020_IMMC%20_Problem.pdf



The tradition of massive flash sales at “brick and mortar” retail stores is spreading around the world. Originally these big sales were scheduled in the

IM²C Funding

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United States to target consumers at the start of a holiday season and on a day they did not go to work (e.g. Black Friday sales). Now, nearly 20 different countries have some form of flash retail sales in attempts to have consumers shop locally and not go online to purchase products.

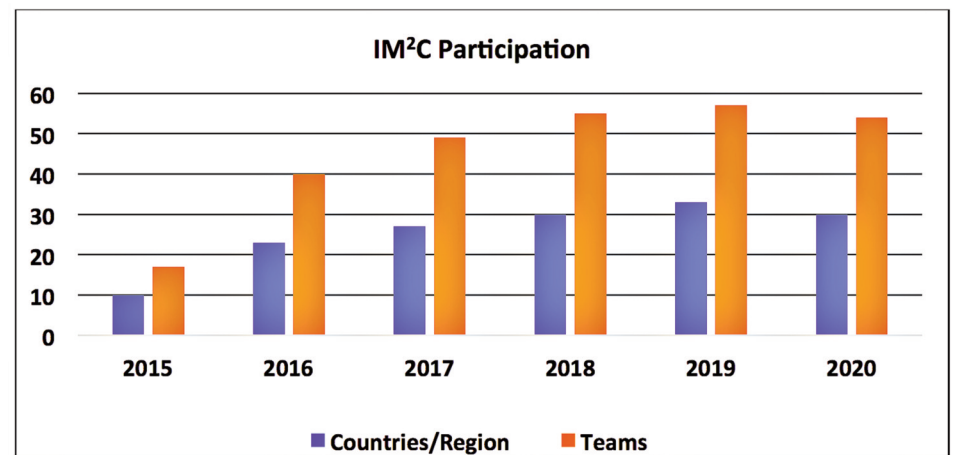
During these flash sales, stores advertise huge discounts on sale items, along with expanded operating hours (usually opening very early in the morning), to attract customers hoping these customers will buy more than just the discounted products.

At the start of these sales, numerous eager shoppers gather at the store entrance to rush in at the store's opening, grab their desired item(s), run to a cashier to quickly make their purchases, and then leave to perhaps travel to another store for more deals. In the process, the over-agitated shoppers are likely to **stampede** and create "human traffic jams," sometimes hurting each other and damaging the surrounding goods. Additionally, some shoppers may not get an item they want due to the item being sold out when they arrive at the item's location in the store.

In preparing for the next flash sale, the manager of an electronic and appliance retail store has asked your team for assistance. The manager is planning a store renovation and has the opportunity to reorganize the layout of the store. The store sells a wide variety of products to include televisions of all sizes, as well as small to large electronics (e.g. cell phones, cameras, home theater equipment) and small to large appliances (e.g. coffee makers, vacuum cleaners, washing machines, gas and electric cooking stoves). The manager has provided you a list of store departments and major categories of products offered in the store, as well as a list of some of the actual items offered during the flash sale to include flash sale pricing, availability, and consumer rating data

The 2020 IM²C Outstanding Teams

School, Location	Advisor	Team Members
Anglo Chinese School (Independent) Singapore	Hong Lee Kiat	Matthew Tham Yong'an Tan Wei Xuan, Keane Jeong Min Lee Ng Jun Kiat, Derek
Canford School The United Kingdom	Henry Bishop	Yuanze Xia Yichen Dong



Numbers of Participating Countries/Regions and Teams 2015-2020

The 2020 IM²C Expert Panel

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The store manager wants to minimize levels of damage to products and believes a store's floor plan and the layout of different departments and categories of products on that floor plan will help to meet these objectives. **Figure 1** provides an example of a possible floor plan.

Requirements

1. The Event.

- Describe the various ways in which products at the store might

be damaged during the sale event due to careless and accidental actions of the customers.

- Consider the items included in this flash sale event. Which sale items do you think will be most popular (most desired) by the shoppers and why?

2. Store Layout.

- Use your responses to Requirement 1 to identify and describe the store layout factors that impact possible dam-

age to products and other measures you deem important during a flash sale event.

- b. Use the factors you identified in part 2.a. to develop a mathematical model or models to quantitatively predict both the behaviors of the flash sale customers that potentially result in damage to products in the store and the level of that damage. Your model(s) should incorporate floor plan characteristics, location of departments, specific flash sale products, and arrangement of cashier stations.
- c. Based on your factors and model, discuss the optimal locations of the store's departments and the most popular/desired sale items. Indicate these locations on the floor plan in **Figure 1**. In other words, label the various areas of the floor plan with your team's choice for the locations of departments and displays of the most popular sale products.

- d. Using your analysis and the model you developed, create and evaluate a new and better floor plan for this flash sale scenario. The store dimensions, scale, location of the entrance/exit, and the items for sale remain the same, but your team can now create its own layout. Justify why your floor plan is better than the one given in **Figure 1**. (Note: Your new floor plan can be hand-drawn or computer created. Include an electronic version or a photo of your floor plan in your submission.)

3. Letter.

Write a one-page letter to the store manager supporting your floor plan layout and discussing any additional strategies for a successful flash sale.

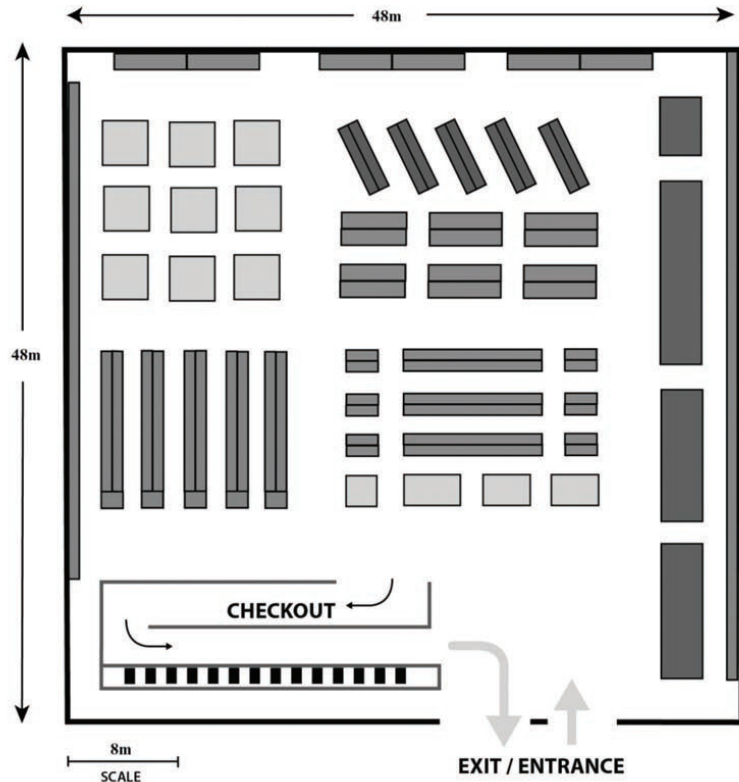


Figure 1. Store Floor Plan

The 2020 International Judge's Commentary

By Frank Giordano

The IM²C judges wish to congratulate all students who took part in IM²C 2020. The judges were impressed by the efforts of all participating teams, the mathematics shown in the solutions, and the high quality of the final submissions. All of the 53 papers submitted (from 30 different countries/regions) showed great creativity in

working on the Challenge. While the Expert Panel judges see only two papers from each participating country or region, we recognize that many more students participate in the Challenge. It is exciting to know that so many students are engaging in, and successfully completing, this mathematical modeling opportunity. We encourage students to continue to form teams, compete to represent their country, and aspire to be named among the top teams at the international level Expert Panel judging.

This year's challenge was to develop a mathematical model to minimize the damage to personnel and goods during a "flash sale" as described in the presentation of the challenge problem above. The "flash sale" scenario was especially challenging as several major issues needed to be resolved before an optimal layout could be constructed and defended. These issues could be resolved either by making assumptions and, if necessary, building submodels based upon the assumptions made.

USA Participation

In the USA, we invite all teams that successfully compete in the HiMCM contest and are awarded a designation of Meritorious or above (Meritorious, Finalist, or Outstanding) to compete in the IM²C. From these participants, U.S. Judges select the two top teams to move on and represent the USA in the IM²C international round. To participate in HiMCM in November 2020, visit www.comap.com.

Among the issues to be resolved are the following:

1. How does damage occur?
2. How do people behave?
3. What are the “popular sale items” to which customers would be attracted?
4. What store layout is optimal for the objective criteria you have identified?

We will describe how some of the teams answered the questions listed above as we address the requirements of the challenge.

Summary Sheet: This should be an overview of the problem and your solution techniques. It should excite the reader sufficiently to want to read your paper in detail. Yes, it is a snapshot of your work but also an invitation to read the paper carefully.

Starting the Modeling Process

Introduce the problem to your audience. From the scenario given, carefully identify the problem you will solve in this paper. Restate the problem identifying all important assumptions that you use to solve the problem you identified. Help the reader understand the tasks and justify (or document) the assumptions you have made.

Restatement of the Problem

Most teams identified a problem similar to “minimize total damage” (personnel and goods) by designing an optimal floor layout that provides optimal pathways based upon product locations which consider:

- The popularity of the goods,
- The crowd behavior based upon the popularity of the goods, and their location,
- The damage to the goods based upon their fragility, location, size, weight, crowd behavior, and other factors,
- An “optimal floor plan” based upon the above considerations.

Thus, the floor layout ultimately recommended requires the consideration of several major issues before deriving a layout that minimizes damage. Let’s now consider how teams answered the above issues.

Popularity of the Sale Items

What affects the popularity of the various products listed? Does the popularity increase or decrease as the quantity on hand decreases? Do customers pay more attention to the amount of the discount (money saved) or the percentage of the discount (good deals)? Are they more interested in the amount saved or getting the “best deal” as measured by the percentage of the discount?

Most teams considered the following ideas or a subset of these ideas:

- The *customer rating* of each sale item as stated in the problem data. While teams recognized that brand names would make a difference in popularity, they assumed the brands were constant due to lack of data.
- The *total amount of the discount* in dollars: the regular price minus the sale price.
- The *percentage of the discount*: the amount saved divided by the regular price.
- The *quantity* of that particular sale item available: Does the popularity of the item increase or decrease as the amount of the item available changes?

Damage Submodel

Among other possibilities, damage to the sale item may have occurred at the production site, in transit from the production site to the store, stocking the item on shelves, during arguments between customers fighting over possession of the sale item, and/or by one or more customers in route to the sales counter. Another important consideration is the fragility of the item. For example, an electric dryer versus a laptop computer.

Most students made assumptions on the above factors. Several teams researched data sites to find indexes of fragility.

Since damage often occurs while the customer transports the chosen sale item from the display area to the checkout area, most teams assumed that the

probability of damage increases in proportion to the distance between the display area of the item and the checkout area. Most teams used the Euclidean distance to measure the distance travelled. Other teams used the more appropriate “Manhattan distance”: the sum of the vertical and horizontal distances between the display area and the checkout area.

Crowd Behavior

Teams used a variety of methods to model crowd behavior. Some teams built on the idea of the sale items creating a “gravitational pull” that would attract customers. Some then identified “heat maps” and “hot spots” based upon the items’ popularity and resultant crowd behavior. To measure the effect of various pathways, some teams treated the customers as a fluid to measure the effect of turning corners and merging with other lanes.

Store Layouts

When the teams had built models to measure the popularity of the items and models to measure the crowd behavior as a function of the popularity of the items, they were then prepared to predict the probability of damage of the various sale items for different layouts. Since the models of damage were probabilistic, most teams built simulations to statistically predict the amount of damage for the current layout and compare that with the predicted damage for several layouts. A few teams built “genetic algorithms” that would reduce the damage estimate in an iterative process. Among the key decisions to be made and tested in their layouts were:

- Where to place the most popular items: near the entrance and checkout or as distant as possible?
- Where to place the fragile items versus the much more stable items?
- How to accommodate the size and weight of the sale items?

Most teams then built layouts and considered various locations of both the entrance and the checkout areas and tested various sale item layouts with their damage model.

Sensitivity

Many assumptions have been made to reach the layout that you recommend to the store manager. Suppose he can't meet all your recommendations in the time and budget available. How sensitive are the conclusions of your model, total damage, to the controllable factors? If there are budgetary problems, which expenditures provide the greatest return for the next dollar spent?

Letter to the Store Manager

At this point teams have used a lot of powerful mathematics to justify their layout which minimizes damage to goods and personnel. The letter should be convincing yet easily understood by the typical store manager.

Results and Recommendations

From the 53 papers judged, two were judged to be Outstanding and two were designated Meritorious. We congratulate these top teams and encourage you to read those 4 papers as they have very different effective ways of modeling the various aspects of this challenging scenario.

In these papers, teams made good choices for the factors considered crucial. The explanation of the factors, and their interdependence, was quite crucial with all four teams doing a good job addressing this aspect of the problem. Also, these teams appropriately justified their choices and uses of particular information, facts, figures, and graphs found on the Internet. This is quite an important issue for the IM²C judging: always justify the information you choose to use and what you do with it! Additionally, teams must document where they found their information by providing a reference to the source.

Finally, the use of appendices is very suitable for both computer code and (big) data sets, since putting these items in appendices helps to increase the readability of the paper itself. Please realize it is hardly likely the Judges will read all computer code in the appendices, so the working of any computer program used must be suc-

cinctly, but fully, explained in the paper itself. Of course, a very important aspect in IM²C is lucid and coherent explanation of the mathematical model that includes the presentation of underlying concepts and assumptions. Teams must make decisions about what to include in the main body of the paper and what to include as supporting material in the appendices.

High-level sophisticated mathematics is not required to make a good model. Teams should make sure they use appropriate mathematics and that they explain their modeling processes and procedures.

Suggestions and Advice for Future Participants

- As stated above, the summary is the reader's introduction to the paper. The summary must not only describe the way the problem is solved, it must also invite the reader to continue to read the full paper. The summary is not the place for complex mathematical descriptions, but instead a general introduction to the problem, your solution process, and your final conclusion.
- As teams have only a short time to do the Challenge they must make assumptions to focus and simplify their work. Teams should make only assumptions that actually impact their model and they should justify their assumptions.
- The most important part of a team's submission is their model. All other parts of their paper support the development, use, and analysis of the model. The mathematics used must always be explained in a logical manner, since this is the heart of modeling!
- Make sure as you model you always stay in touch with reality. Since the problem is a real-life-problem, it is quite essential that teams reflect and critically judge their mathematical solution as calculated.
- Since even a good model cannot be perfect, especially when developed

in only five days of work, an analysis of the **strengths and weaknesses is required.**

- Ensure that your paper concludes with a short summary of the actual solution or findings to the requirements of the problem.
- With only 20 pages to introduce the problem, state assumptions and justifications, develop, solve, and apply the model, and do some analysis to include identifying strengths and weaknesses, there is not much room for tables of data or code. While you might include a short section of code or a small subsection of a data spreadsheet, computer code and (big) data sets should not be in the paper itself but in an appendix.
- Teams must properly document and reference all information taken from the Internet or other sources, including graphs, illustrations and pictures. Teams can use in-line documentation, footnotes, or endnotes and also include a Reference List.
- Teams should also take notice of the page limitation of 20 pages, and the rules for font size and margins.

Finally, the IM²C judges would like to compliment all teams on their effort, and thank them and their teachers/advisors for joining IM²C 2020. All teams did a great job in diving into a very complex problem for five days. The result was quite a number of very creative papers. The judges (all mathematicians and teachers) had some stimulating discussions about the papers, as it was clear that the students gave some very thought provoking analyses. This shows that in many countries and schools, mathematical modeling is a growing field of interest that students enjoy and are very capable of doing. Well Done!

General Advice to Teams Participating in Future IM²C

The IM²C is definitely a challenge. Teams have to organize themselves,

address all requirements of the problem, and write a report all in a short period of time. Budgeting time becomes critical so that you leave enough time to effectively communicate your work and results to the challenge judges.

Our advice is to allow plenty of time to construct your report. In fact, consider outlining the report as soon as you begin working on the problem. This outline will guide your team in its work and provide a logical path to your solution for readers of your report. Remember, you are communicating with judges from many countries of the world. The judges are not necessarily familiar with the curricula of your school, so present the development of your model in a logical and easily understood fashion. Judges are not looking for the papers that use the most sophisticated mathematics. Do not force the mathematics upon a given scenario. Rather, begin with the simplest mathematics that solves the problem you have identified. Later, as appropriate, refine and enhance your model to increase its precision, or adjust your assumptions to find a more broadly appropriate solution.

Pictures, graphs, tables, and schedules can be quite effective and efficient in communicating your ideas. The use of relevant pictures and graphs can make a report clearer and more pleasant to read. Your report should include a combination of various representations: symbolic, graphical, and text that best present your model and solution. Realize, however, that large tables and extensive code or data might be better as supporting material located in an appendix.

The use of symbolic formulae and algorithms are quite essential in a mathematical modeling assignment. The use of unexplained formulae, however, will not make the report more convincing. The reader needs to believe that the writers themselves understand the formulae used. This is

done through explanations and analyses of your modeling processes. The readers of your report, while experienced mathematicians, are not experts in all parts of the great world of math!

Appendices are very useful, but do not expect the judges to read them. While judges may refer to an appendix to check a reference or to get a general idea of your computer code, they will not fully read the appendices. Therefore, do not place anything critically important to the development of your model in an appendix.

Remember to list any sources you used during your work on the Challenge and to document in your paper where you used these sources (e.g. a graph or picture from a particular web site). Follow the rules of completing your solution report within the specified number of pages and in a font size of no smaller than 12-point type.

Overall, present the development and analysis of your model in a manner that a wide audience could understand. Consider who might be using your model and explain your model to that audience, as well as the judges. Ensure you close your report with a conclusion and a summary of your results.

**For more information about the
IM²C, including the complete
2015–2020 results and
sample papers, visit
www.immchallenge.org**

Flash Sale

Anglo Chinese School (Independent) Singapore

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Jeong Min Lee

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1 Summary

Flash sales are increasingly becoming the norm for brick-and-mortar stores, mainly in an effort to redirect sales away from online stores. These strategies cause surges of shoppers during these sales, resulting in many accidents that both damage products and injure people as they rush to purchase their target goods. The importance of store layouts to minimize the degree of such damage in stores has thus emerged, like in this problem.

For our model, we sought to minimise the financial losses to the store from these item damages, referred to as the **total loss, the dependent variable**. We first identified the main factors in the surge of shoppers: the positions of the most desirable goods, the cashiers, and the presence of bottlenecking physical elements. The desirability of the goods was assessed based on an index coined the **Gross Happiness Index (GHI)**, which took into account discounts, consumer rating and quantity remaining of each product. This formed the basis of our model's functionality.

The layouts were converted into a 48x48 matrix to simplify computations, with each tile representing a 1m by 1m square.

We then explored various ways in which consumers may damage items, such as knocking them out of their shelves, fighting for the last remaining stocks and walking through the crowd with the items. We realised that these were all related to **shopper density** in the store. We thus derived a formula relating **shopper density, item fragility, number of unsold goods and the discounted price of the item** to obtain the monetary loss of one tile resulting from the damage of items. The sum of monetary loss of all the tiles in the store was taken as the **total loss**.

To simulate the passage of shoppers through the various layouts, we used the a* pathfinding algorithm to construct an agent-based model. The likelihood of a good being the desired good of each shopper was based on the GHI of the good. Each agent, representing an individual shopper, would go from the entrance to the shelf with the desired good, the cashiers, and finally the exit. The various paths were then merged onto one matrix to find the shopper density per tile and thus the total monetary loss.

A modular function was created to test large numbers of different item arrangements to arrive at the optimum item arrangement based on GHI, which was determined to be one with **decreasing GHI from the cashier**. The optimum “prominence” threshold (**0.9**), which indicated the range of popular goods which should be displayed separately from their department to minimise total loss, was also obtained.

Different floor plans were tested using these metrics. The **optimal item arrangement** from above was used in **different layouts** hand-created by the team. Without changing the size and shapes of shelves given, an optimal layout was found. However, given liberty to modify shelves completely, the grid layout format was then determined as the most optimal layout type and variants of it were tested to select the **optimum layout** which minimised total loss.

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3 Introduction

Black Friday is an annual tradition that has been held in the US for decades (Pruitt, 2015). Held the day after Thanksgiving, the holiday sees retailers slashing prices on many products in a bid to turn a profit. Consequently, many Americans hit the shops on Black Friday every year, with 135.8 million Americans indicating that they would spend the day shopping in 2015 (Pruitt, 2015). However, this rush for goods has resulted in many accidents, injuring over 100 people from 2006 to 2018 (Crockett, 2019). Not only has this resulted in injuries, but it has also resulted in the damage of many consumer goods, either due to shoppers knocking over goods through carelessness or through consumers fighting over them (Katz, 2019). Due to shoppers rushing to obtain a particular good and scrambling to pay for it so as to save time to buy more goods from other retailers, stampedes may even occur, causing injuries and damage to goods. Thus, Black Friday has caused many producers to suffer losses instead of turning over profits.

Furthermore, in this digital age, more and more consumers are turning to online shops like Amazon instead due to reasons such as convenience. As a result, more brick-and-mortar shops are implementing flash sales in a bid to attract customers to purchase from them instead of from online department stores, increasing the risk of suffering losses due to damaged goods as a result of an uncontrollable influx of consumers.

Therefore, it has become increasingly essential to manage such instances of ‘human traffic’ so as to ensure that the endangered brick-and-mortar shops can achieve the intended effect of turning a profit rather than suffering losses. The model developed by our team aids this by manipulating store layout and product distribution to redirect shoppers in a manner as to reduce the number of accidents.

Although similar studies have been done in the past where methods like Dijkstra’s algorithm and Bellman-Ford’s algorithm were used to find the shortest path to obtain goods in a store (Dela Cruz et al., 2016), the model developed by our team uses a novel method as it involves manipulating both the store layout and the customer path, which enhances its effectiveness.

4 Model

4.1 Operationalization of Consumer Preferences

Examining the store data provided, we identified the discount offered, consumer rating, and quantity available as factors likely to affect consumer preferences toward an item.

4.1.1 Discount offered

A consumer’s perception of the “value” of a discount is based on two factors: the absolute monetary value of the discount (i.e. price – discounted price) and the percentage discount (i.e. $\frac{\text{price} - \text{discounted price}}{\text{price}} \times 100\%$). Intuitively, as the discount amount and percentage discount increases, the consumers’ perceived gain from purchasing that good would increase as they save more money from the original price. We hence “double-count”

the discount in our modelling of consumer preferences by taking into account both discount amount and percentage—this is necessary to reflect consumers' tendency to place more significant weightage on the extent of the discount offered than other factors in the context of a flash sale situation. This is corroborated by Fam et al. (2019) who found that 'discounts and coupons are the two most highly ranked SP (sales promotion methods) across the sampled countries' in their study.

4.1.2 Consumer rating

Consumer rating is taken as a baseline measure of product attractiveness, which represents how consumers see an item regardless of whether it is on sale. Accordingly, consumers are more likely to buy a more highly rated item.

4.1.3 Quantity available

Conversely, the lower the quantity of an item offered, the higher its perceived rarity. As consumers are more likely to perceive their purchase as a better deal if they are among the few able to get the item, goods with a lower stock will be more attractive to consumers. This is a valid relationship. In fact, this is why many consumers are willing to queue up for hours in harsh conditions just to purchase limited edition products from US brand Supreme (Clifton, 2016).

4.1.4 Gross Happiness Index (GHI)

Taking into account all three factors above, we defined a Gross Happiness Index (GHI), a measure of the total pleasure or utility derived by consumers from successfully purchasing one unit of a given product. GHI was calculated with the following formula:

$$\text{GHI}(\text{item}) = \frac{\text{amount discounted} \times \text{percentage discount} \times \text{consumer rating}}{\text{quantity available}}$$

To verify that our operationalization is reasonable, we examined the GHI of a number of items in the provided dataset. The most popular goods were the 5.3cu ft Slide-In Electric Range, Stainless Steel (GHI = 212), the 30" Combination Double Electric Convection Wall Oven with Built-In Microwave (GHI = 330), and the 24.7cu ft French Door Refrigerator, Black Stainless Steel (GHI = 398).

This index could alternatively be interpreted as the relative desirability of an item—as such, we calculated the probability that someone will aim to buy a specific item as:

$$P(\text{buying a particular item}) = \frac{\text{GHI}(\text{of that item})}{\sum \text{GHI}}$$

4.2 Encoding Schema for Store Layout

4.2.1 Fundamentals

In order to run a simulation of customer behaviour in a store, we had to encode its layout and item positions in a format which can be interacted with programmatically. As the problem statement provides us with a 48m × 48m floor plan, we chose to use a 48x48 grid as the environment for our simulation, where each grid unit had dimensions of 1m by 1m.

In order to faithfully recreate the given store layout, we extracted the original image and converted the picture to black and white, removing all annotations and arrows. We then downscaled the image to a 48x48 pixel bitmap (.pbm) and inverted the colours. This process is shown in Figure 1.

With reference to Figure 1, the leftmost image is the original given image of the layout, the centre image is the image in black and white, and the rightmost image is the image converted to a bitmap.

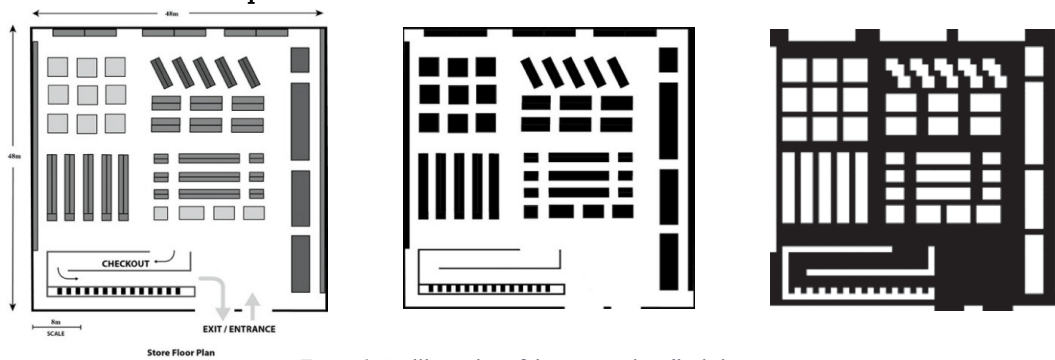


Figure 1: An illustration of the process described above.

4.2.2 Layout components

Hereafter, we refer to each pixel in the bitmap as a *tile*, representing a 1x1m area of the store. 8 different tile types were identified, and each assigned a unique colour. Our final encoding scheme for a store’s layout is displayed below:

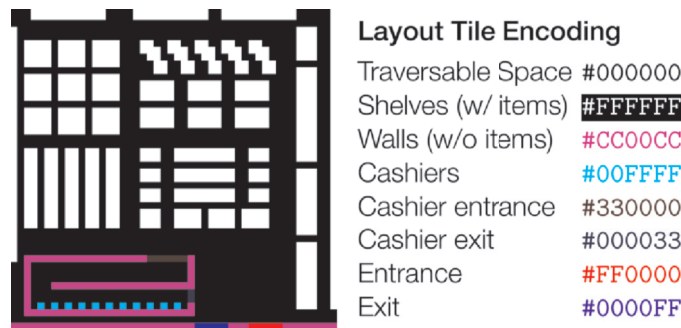


Figure 2: a legend of the tile colours in the bitmap

With reference to Figure 2, *traversable spaces* refer to the black tiles. These are the areas where shoppers can move. They include empty spaces and corridors between shelves. The *shelves* are illustrated as the white tiles where items are placed, and the *walls* are the purple tiles. They are just walls to divide/block shoppers from passing through, and no items will be placed on them. We made this distinction as large items like those listed in the provided spreadsheet (e.g. 85” TVs) are rarely placed along cashier queues or other

inconvenient locations. This also provides us with a utility to manually influence the placement of items—though this was generally performed programmatically, as will be elaborated upon later. *Cashiers* are illustrated as blue tiles, where people make their payments. The *cashier entrance* is illustrated by the maroon tiles, and the *cashier exit* is illustrated by the dark blue tiles. This allows us to optionally indicate where a queue for a cashier will start and end. The rationale for needing to define queues in this manner explicitly will also be elaborated upon further in later parts of the report. Finally, the entrance and exit to the store are the bright red and bright blue tiles respectively.

4.2.3 Data representation

Our custom simulation software imports a bitmap of a store layout via the function `read_img_map()`, separating each set of tiles of a given colour into its own 48x48 matrix, as shown in Figure 3. By primarily dealing with the store's layout as a set of matrices, most computations in our simulation are reduced to matrix operations, maximizing efficiency and minimizing simulation runtime. This is especially important due to the large number of layouts which need to be tested.

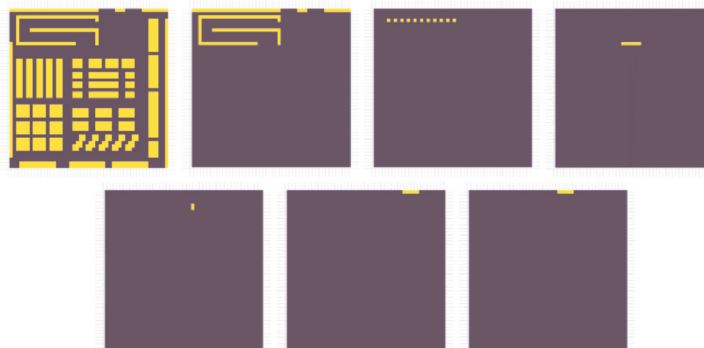


Figure 3: Internal matrices

4.3 Simulating Consumer Behaviour

4.3.1 Behavioural Assumptions

A number of assumptions about shopper behaviour were made, which informed the design of our models and simulation.

4.3.1.1 Assumptions with regard to desired items

Firstly, we assumed that consumers desired only one good and knew what the desired good was before entering the store. This also meant that the consumers did not impulse buy goods other than the originally desired good. This is not an unreasonable assumption, given that considering how crowded retail stores can be during retail sales, consumers will often strategise to target a specific item of interest, and rarely stick around after purchasing their desired good. This was a necessary assumption in simplifying the movement of the agents towards the desired shelf.

Secondly, it was assumed that the shoppers have no prior knowledge of the item positions

in the store. This is a valid assumption, considering that the store's layout is changed just before the flash sale begins. Hence, this meant that the shoppers needed to perform a search through the store to find their desired item. We used a*star search as an approximation of human pathfinding.

Thirdly, we assumed that consumers do not know if the good that they wanted was already out of stock before they entered the store. Even if a particular good is already out of stock, consumers would still make their way towards the particular aisle. This is also a valid assumption—this is the underlying reason why fights occur in the first place, as shoppers try to snatch goods from other people. This was a necessary assumption because it allowed us to simplify the code involved in modelling the behaviour of the agents.

4.3.1.2 Assumptions with regard to other shoppers

As shoppers have physical volume, they are unlikely to fit into the same region of space. This phenomenon is responsible for congestion and jam formation as shoppers try to squeeze past each other while moving through corridors, entrances and exits, drastically slowing movement in those areas. Hence, shoppers would, as far as possible, avoid these regions of congestion so as to get to their desired items quickly. This was a necessary assumption because it improves the accuracy of the model by more realistically simulating the behaviour of shoppers in real life.

4.3.1.3 Assumptions with regard to item damage

One final assumption is that the frequency of accidents and fights between consumers is assumed to follow a power law with respect to the density of people. This is because the more packed a particular walkway is, the more likely that shoppers would accidentally knock over a particular good due to carelessness. Moreover, the more crowded a particular aisle is, the more people there would be fighting over a particular good, hence the higher the frequency of fights occurring. These probabilities would only start increasing in extremely packed situations, because in moderately packed situations the likelihood of these accidents is still reasonably low—for example, if only 10 people walked past an item a day, the probability of an accident is negligible. Without a doubt, the frequency of fights is also dependent on factors like the temperament and personalities of the shoppers. However, it was not possible to take into consideration these subjective factors when designing the model; besides, as the model intends to estimate the average loss, these variables are unlikely to affect the final estimated loss in any case drastically.

4.3.2 Simulation Principles

4.3.2.1 Fundamentals

We adopted an agent-based approach for behavioural simulation, with parameters chosen so that each agent roughly represented 10 real-world shoppers. Each agent followed a simple set of rules based on the aforementioned assumptions of shopper behaviour, as described below:

4.3.2.2 In-store Pathfinding

Each agent was initialized with a stored list of target coordinates to a specified entrance tile, desired item tile, cashier tile, and exit tile. The likelihood of a good being selected as the desired item was based on its GHI, as described above, while specific entrance, cashier, and exit tiles were selected from their respective matrices with equal likelihood.

Agents travelled between these tiles in a set sequence (i.e. entrance→target item→cashier→exit), with routes between the tiles calculated via the a* pathfinding algorithm. Briefly, a* is a graph traversal algorithm which performs a heuristic search for the shortest path between nodes. It saves time by only checking a limited set of adjacent nodes (the “open set”) to the current node at each step: for each of these nodes, it estimates the total path cost by summing the distance travelled to that node and the expected remaining distance to the target, as calculated by a distance heuristic (in this case, the Euclidean distance between a node and the target node), then designating the node with the lowest total path cost as the new current node.

We chose a* search as it is a better approximation of a human search for the shortest path (where we are limited by possible paths within our lines of sight) and time-optimality compared to other search algorithms, such as Dijkstra’s or Bellman-Ford.

4.3.3 Crowd Avoidance

This process also meant that individual agents did not interact in a shared world every simulation tick; instead, interactions were iteratively computed between agents’ complete paths through the shop, making the model “approximate”.

To compute these “interactions”, a shared density matrix was instantiated, and the drawing of paths commenced. Every time an individual path included a tile, that tile’s density was incremented by 1. As previously assumed, people seek out areas of lower pedestrian density while searching for a path through an environment. Therefore, we defined an avoidance coefficient (A) based on the density in a tile. We used this coefficient to increase the distance to tiles with higher densities. For example, the distance to an adjacent tile of 0 density is simply the Euclidean distance (1), but the distance to an adjacent tile of density 1 would be the Euclidean distance + density × A, which would be (1+A). Hence, this coefficient has the effect of artificially increasing the distance travelled in more crowded routes, so that the agents would avoid them and take less crowded paths instead. We used A=0.1 for all our simulations. While the avoidance coefficient was arbitrarily determined, it does have a physical meaning—the value chosen means that an agent density of 10 at a tile would make it twice as difficult to get to that tile.

4.4 Estimating Monetary Loss

4.4.1 Causes

There are various ways consumers may damage goods during flash sales. As crowds of shoppers are pushed along the store in an effort to rush to their purchases, some shoppers may be pushed towards the shelves to knock down small products on the shelves,

damaging them. Larger products placed on the floor may also be knocked down or squashed between crowds to be damaged.

Products may also be damaged during the shoppers' efforts to acquire them. Removing goods from the shelves in the middle of such a crowd would be difficult and may result in the goods being dropped onto the floor. Fights may also occur between shoppers as they try to acquire the limited stock of goods, pulling on the goods and damaging them.

After the shoppers acquire the good, the good may be damaged as they try to wedge out of the crowd, squeezing against the good. Shoppers holding large products are especially prone to this as they may lose balance, injuring themselves and damaging the product.

Furthermore, in a sale situation where desirable goods are in limited quantity, fights may occur due to multiple consumers believing that they have the right to buy a particular unit. This can further cause damage to units being fought over as they may be roughly handled during a scuffle. In addition, if a fight were to break out amongst cramped shelves, the products on the shelves may be knocked off, causing damage.

Evidently, the above cases are more likely to occur as there are more shoppers, represented by a higher human density. As stated above, the frequency of cases would only increase in extremely packed situations. Hence, the function for the amount of damage inflicted is nonlinear with respect to density, with the amount of damage taken to be proportional to the human density squared.

$$\text{Amount of damage inflicted} = \text{density}^2$$

4.4.2 Factors Affecting Loss

Thus, we assumed that the monetary loss for a tile was a function of the amount of damage inflicted in that tile and the properties of items placed nearby the tile.

In all, the estimated loss due to the damage to an item from pedestrian density in a given tile was a function of the amount of damage inflicted, the item's unit monetary value, a decay term taking into account the attractiveness and quantity of the item, and the item's fragility.

The unit monetary value was taken to be the discounted price, as it would be the amount of money received by the store if the good was undamaged. The decay term was taken into account because the quantity of an item remaining will decrease at a higher rate if it is more sought after.

4.4.3 Item Fragility

Fragile goods are more likely to be damaged to a more severe extent, resulting in more significant monetary losses. We defined a fragility index, F_1 , to represent the likelihood of an item being damaged in the store. This was calculated using real-life data of the percentage of the product damaged in shipping (Blumberg, 2005).

$$F_1(\text{item}) = \% \text{ of items returned due to shipping damage} \times \% \text{ of all items returned}$$

F_I can be understood as the probability that an item would be damaged as a result of being shipped once. Therefore, we need to relate the amount of damage inflicted due to pedestrian density to the “number of times shipped”. We make an educated guess that 1000 people walking within 1 metre of an item on a busy day will cause an equivalent amount of damage to a product being shipped 10 times. We use this guess to define a constant, c , relating the arbitrarily defined amount of damage inflicted to our fragility index and “number of times shipped”:

$$\frac{1000^2}{c} = 10$$

$$c = 100000$$

With this constant, we define a density index, D_I , calculated:

$$D_I(\text{tile}) = \frac{\text{density}^2}{100000}$$

4.4.4 Total Expected Loss

The total monetary loss due to density at a particular tile was equal to the summation of monetary loss to items in the adjacent tiles.

To find this monetary loss to items, we used the following formula:

$$\begin{aligned} &\text{Monetary loss to item in adjacent tile} \\ &= (D_i \times F_i) \times \text{discounted price of item} \times \int_0^1 \text{qty of item } dt \end{aligned}$$

Where t is the proportion of the day passed.

If items are still remaining after the sale,

$$\int_0^1 \text{qty of item } dt = \text{remaining qty} + \frac{1}{2} (\text{initial qty} \times (\text{initial qty} - \text{remaining qty}))$$

Where

$$\text{remaining qty} = \text{item qty} - \text{expected purchased qty}$$

And

$$\text{expected purchased qty} = P(\text{item purchase}) \times \text{number of agents}$$

If items are all bought,

$$\int_0^1 \text{qty of item } dt = \frac{1}{2} (\text{initial qty} \times \text{time sold out})$$

Where

$$\text{time sold out} = \frac{\text{initial qty}}{\text{expected purchased qty}}$$

With these operations, we could evaluate the total monetary loss per tile, which could be summed to obtain the estimated monetary loss over the entire store. Hence, we had an operation that maps a matrix of pedestrian density to a matrix of estimated monetary loss.

4.5 Model Evaluation

4.5.1 Validation Against Real-world Behaviour

We found that our model could successfully simulate many observed pedestrian behaviours, including bottlenecking at tight corridors and jamming at intersections. An example with random item layout is shown below, demonstrating these characteristics.

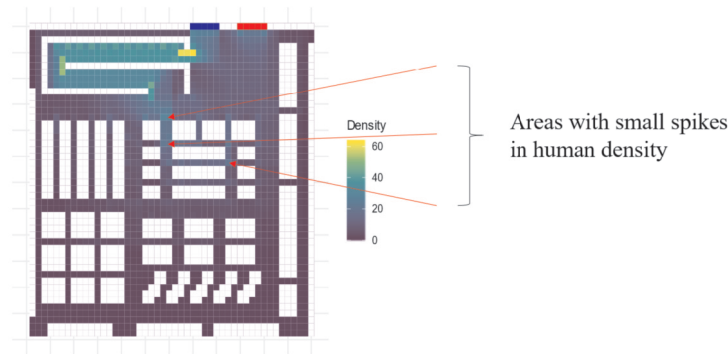


Figure 4: running our simulation on the given layout with randomised item arrangement

With reference to Figure 4, the legend indicates that dark blue areas have the lowest agent density, followed by green areas and yellow areas. The green colour of the walkways along the cashiers shows the high density of agents in the narrow corridor, especially at the even narrower exit. Smaller spikes in human density can also be seen in the intersections of the walkways between shelves, as labelled by the red arrows in Figure 4.

The simulation also produced results that matched intuitions about item positioning. As stated above, if popular items are placed near the entrance, exit, and cashier, the paths to obtain these goods would be shorter, resulting in the decrease of total population density from these paths and thus decreasing total loss. This is evidenced by Figure 5. The simulation was run twice: once for a layout where high GHI goods are placed near the entrance, exit and cashiers, and another time for a layout where low GHI goods are placed near the entrance, exit and cashiers. The results of the simulations are shown in Figure 5.

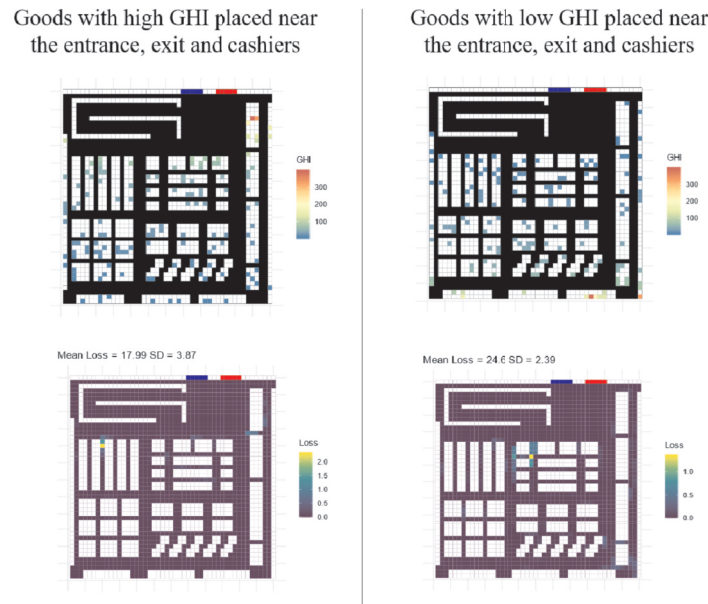


Figure 5: a comparison of simulations run for two different arrangements of goods.

The left side of Figure 5 is the simulation run for the layout where goods with high GHI are placed close to the entrance, exit and cashiers, while the right side is the same results for the low GHI layout. The top half shows the arrangement of the goods with different GHIs, and the bottom half shows the monetary loss. As can be seen, the top left image shows that high GHI items (red) are placed close to the entrance, followed by low GHI items (blue) placed further away. The opposite is true for the top right figure. Consequently, the bottom left image shows that the average monetary loss for the layout where high GHI goods are placed close to the entrance is 17.99, and the average monetary loss for the low GHI layout is 24.6, which is considerably higher.

Additionally, as we were trying to calculate a cumulative loss value, ultimately, the vast majority of time-varying behaviour did not benefit analysis and was computationally wasteful to simulate. For example, a simulation encompassing oscillation of passing direction at a bottleneck will average out to a diffuse region of high density around the bottleneck, producing the same result as our model. The model hence essentially simplified this process to save on computational power.

4.5.2 Robustness

However, since the simulation is iterative, results technically depend on agent order, though this effect likely diminishes with larger agent numbers. Furthermore, the item positions on each shelf were also randomized for each replicate, introducing further variance to the results of the model. This made the model stochastic as the results were heavily influenced by random factors.

To account for this effect, we ran 10 replicates of each simulation each with randomized agent orders and item positions. This would have normalized the variations of the results stemming from the agent orders. Fortunately, this barely hindered computation, as the operation is fully parallelizable across multiple CPU cores. On top of this, we computed the standard deviation for the 10 replicates, allowing the comparison of preciseness between layouts and their corresponding item arrangements. The stochasticity was thus overcome to some degree, making the model robust.

4.5.3 Limitations

Despite the strengths of our model, every model has its limitations.

Firstly, the fragility index formulated by our team may not be completely accurate. This is because the fragility index was calculated based on shipping data, provided by Blumberg (2005). The data provided statistics about the average percentage of a particular product type getting damaged in shipping. For instance, according to the book, on average, 2.36% of all desktop computers get damaged in shipping. This percentage was used as an indication of the fragility of the good since goods which are more likely to get damaged during shipping are likely more fragile. However, this data has two main shortcomings. Firstly, according to the book, on average, 2.232% of all televisions get damaged during shipping. However, this percentage does not make a distinction between different types and sizes of televisions, while the store inventory provided includes televisions with sizes ranging from 30 inches to 85 inches. Since the data provided does not make a distinction between them, they were all assumed to have the same fragility index. This is not entirely accurate, since size may play a part in affecting the fragility of the item.

Another limitation of the data is that it does not include the statistics for certain goods. For instance, the average percentage of goods getting damaged in shipping was not provided for robot vacuums and gaming consoles, both of which appear in the store inventory. However, the data does provide such statistics for all consumer electronics and major appliances on average. Major appliances referred to necessities like refrigerators and washers, while consumer electronics generally referred to entertainment products like laptops and televisions. Hence, for items on the store inventory whose data was not provided, average data for all major appliances/consumer electronics had to be used, depending on the particular good. This could have resulted in inaccuracy since the actual average percentage of robot vacuums (for example) getting damaged in shipping may be different from the average percentage of all appliances getting damaged in shipping.

These two limitations mentioned above can be mitigated by referring to a variety of sources in determining the fragility index. However, due to the short period available in coming up with the improved layout, there was insufficient time to carry out a meta-review of real-life statistics.

Another limitation arises due to the inability of the layout/model in taking into account subjective factors. As mentioned above, the likelihood of damage also depends on the personalities of the customers, as short-tempered consumers are more likely to get into fights/arguments, which may lead to damaged goods. However, such subjective emotions cannot be modelled via mathematical means; thus, this is an inevitable limitation.

One other limitation of the layout/model is that it does not take into account the different brands of the goods on sale. According to the store inventory provided, the items on sale are manufactured by different brands like Brand FF, Brand M, Brand W etc. The branding will likely have a significant impact on the gross happiness index (GHI) because consumers will likely derive more pleasure when they purchase a brand that is more famous and well known, which, as mentioned above, is why many consumers go after Supreme products (Clifton, 2016). However, the different types of branding were not taken into account in formulating the GHI, because of the simple reason that based on the brand names provided, the more popular brands could not be distinguished from the less popular brands, and the significance of branding could not be factored into the formula for the GHI.

5 Model-guided Design

5.1 Approaches

5.1.1 Computational

Using software to randomise the layout of the shelves was considered. However, doing so was determined to be extremely time-consuming. This was because the total number of shelves and checkout stations was 53. Even accounting for shelves with the same dimensions, the total number of ways to permute their physical locations in space was still an extremely large number (order of magnitude ~ 40). Furthermore, other than the location of the different shelves, other factors also came into play like whether the shelves are positioned horizontally, vertically or at various angles, further increasing the total number of ways to arrange them. Machine learning approaches—genetic algorithms and simulated annealing among them—were considered, but ultimately abandoned due to massive size of the search space.

In the end, a modular function was created to allow for different organisational methods to be used for item placement within the space and item organisation based on GHI and fragility. The modularity of this function allowed for the addition differ

5.1.2 Human-guided

Thus, our team decided to manually design layouts which we deemed suitable for a flash sale, iterating upon them based on simulation results. This was possible as we were able to edit the layouts directly with an image editor rather than editing every value of the matrix, allowing for rapid creations of layouts.

We researched a list of commonly used store layouts, such as the forced-path layout, the grid layout, the geometric layout etc. (“Retail Store Layout Design and Planning”, n.d.).

Some of these layouts were deemed to be not optimal for flash sales, as seen below. We decided that the grid layout was the best for flash sales and designed floor plans around this layout. The matrix model mentioned was then used to test which layout allowed for the lowest likelihood and extent of damage.

The various layouts available for design are as follows:

Forced-path layouts are the layouts widely used by Dutch furniture retail giant IKEA where consumers are forced to move through the entire store. Such a layout is not optimal for the flash sale since consumers generally have something they desire and would not want to waste time moving around the entire store. An example of a forced-path layout is shown in Figure 6.

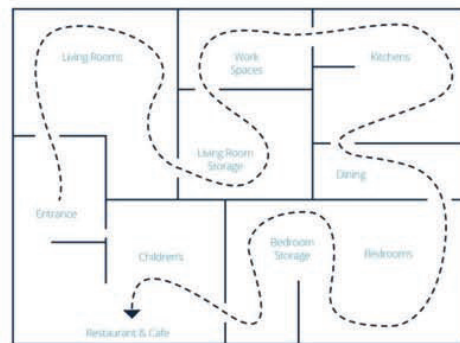


Figure 6: An example of a forced-path layout. Retrieved from "Retail Store Layout Design and Planning", n.d.

Similarly, loop layouts, which force customers to go one entire loop around the store, are also not feasible for the same reason.

Diagonal layouts where the shelves are arranged diagonally are also not optimal for flash sales, because arranging the shelves diagonally would result in narrower aisles which lead to an increased likelihood of damage. The given layout does include a row of diagonally arranged shelves. An example of such a layout is shown in Figure 7.

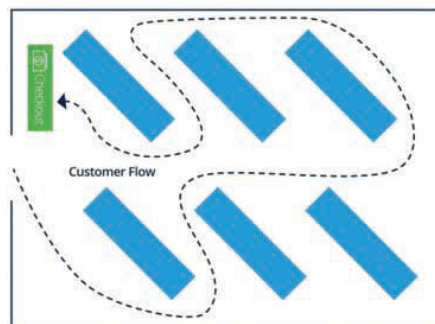


Figure 7: An example of a diagonal layout. Retrieved from "Retail Store Layout Design and Planning", n.d.

Grid layouts were deemed to be optimal for flash sales because it allows for more efficient movement of customers and greater distance between shelves allowing for reduced likelihood of damage. The only downside of the grid layout is the lack of visual and aesthetic appeal. However, this is generally not a consideration during a flash sale. An example of a grid layout is shown in Figure 8.

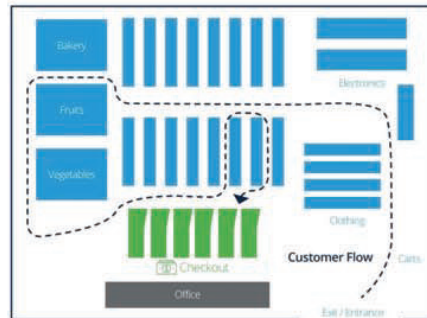


Figure 8: An example of a grid layout. Retrieved from "Retail Store Layout Design and Planning", n.d.

Various grid layouts were designed based on the principle that the most popular items with the highest GHI should have the greatest spacing around them, so as to accommodate higher numbers of agents (shoppers) and hence minimize human density and thus item damage.

5.2 Experiments (Note: in each experiment, the dependent variable was the monetary loss)

5.2.1 Independent Variable: Item Placement in Original Layout

In this section, the variable tested was the different methods of arranging items given their different GHIs on the default shelves provided.

Our baseline was a random initialisation of the shelf and item positions on the original layout, which resulted in a loss of 17.83. Visualisations of the layout generated, pedestrian density, and loss are provided in Figure 9.

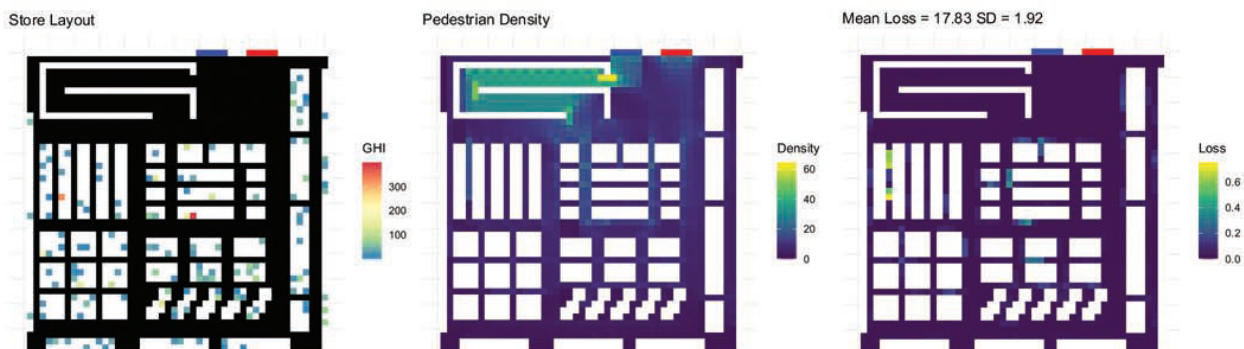


Figure 9: Maps of random initialisation of item arrangement

Using the mathematical model, the shelves of products were rearranged to yield the lowest total loss. We refined this arrangement further to place the items with higher GHI closer to the cashier (in terms of Euclidean distance), and items with lowest GHI further away from the cashier. This further lowered the monetary loss to 11.69.

Placing the high GHI goods closer to the cashier allowed the minimisation of total loss as the paths taken by the shoppers, which involved moving from the good to the cashiers, decreased in length. Thus, the total density contributed by these paths decreased, leading to the decrease in total density in the store and hence the decrease in total loss, which is visible when comparing Figure 10 to Figure 9.

Up to this point, item placement has not taken into consideration product category, as this factor is not taken into account in the agent pathfinding—that is, agents will find a path to an item just as easily, whether or not they are sorted into departments or not. Of course, actual shoppers would find it much easier to search for items according to category. Therefore, any practical store layout would only place a limited number of high GHI items in individual shelves, while all other items would be sorted by department for easier access.

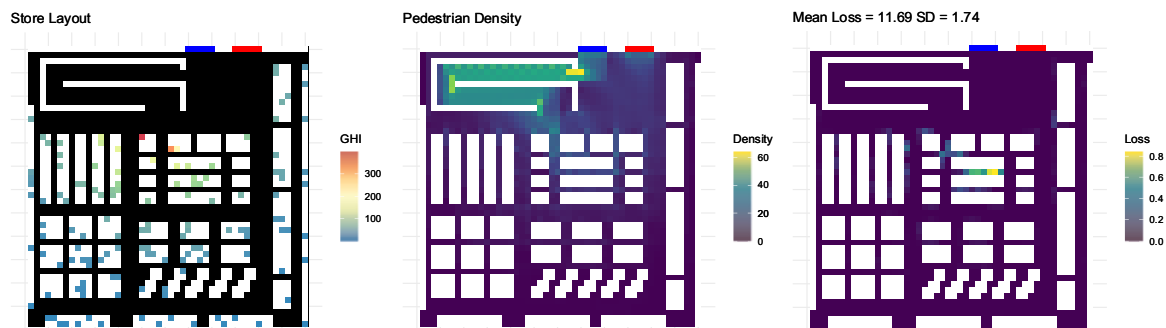


Figure 10: Maps of item arrangement of decreasing GHI from cashier

In our code, we accounted for this factor by determining a prominence threshold, P , representing the percentile of GHI above which we consider an item popular enough to be assigned its own display. First, items above that threshold were placed in positions closer to the cashier, in terms of decreasing GHI, regardless of category. Items were then assigned positions according to their category, with their distance from the cashier determined by the category's average GHI. Positions of items within their own category area were further sorted by item GHI. An illustration of this sorting procedure is shown below:

Prominence thresholds ranging from 0 to 0.9 were tested, with 0.9 being the optimal “prominence threshold” value, resulting in the lowest loss value of 11.2. A graph of loss against threshold is displayed in Figure 11.

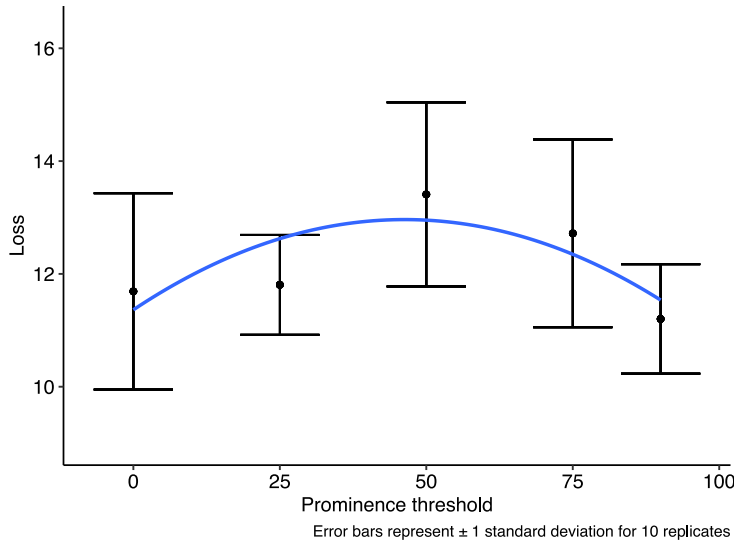


Figure 11: Prominence against loss

As seen in Figure 11, although a trend is observed, the large error bars indicate that it may not be significant. However, this is ideal as it indicates that sorting by department does not greatly affect the loss, allowing for easier identification by customers when they visit the store.

5.2.2 New possible layouts

5.2.2.1 Independent Variable: arrangement of shelves

Many designs were made by rearranging the given shelves (without changing the dimensions), and testing was then done on the layouts. The algorithm for item arrangement followed that from section 4.2.1, with a prominence threshold value of 0.9 and an arrangement of decreasing GHI from the cashier. 5 generated layouts and their associated loss values are presented below in Figure 12:



Figure 12: Maps of possible arrangements of shelves

The pedestrian heat-map of the final layout can be found in Figure 12. A spike in pedestrian density around the corner of the walls of the walkway and funnelling through the walkway can be seen.

A mean loss of 9.71 and a standard deviation of 0.74 was recorded as seen in the last map of Figure 13, the bottommost map, significantly lower than the mean loss of 11.69 of the original layout as shown in Figure 10.

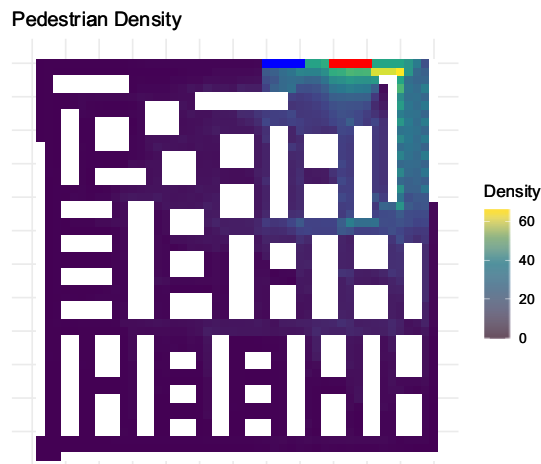


Figure 13: Pedestrian density heat-map of final layout

With reference to Figure 10, for the same item arrangement on the given layout, the monetary loss was 11.69. However, as can be seen in Figure 12, the monetary loss for the chosen final layout was only 9.71, which is objectively a lot lower than that of the given layout. This can be explained with various factors. Firstly, in the given layout, there is very little space in between each shelf, with most of the corridor widths being as narrow as 1 metre. In contrast, with reference to Figure 12, there is at the very least a 2 metre spacing between each shelf, with some spacings being as wide as 3 or 4

metres. This helps to greatly reduce the human density around the items, which is especially important for the more fragile items. In addition, another main difference lies in the fact that in the original layout, there are not many paths which consumers can take. For instance, when consumers are trapped in between two long shelves, they only have two available paths, which results in a high human density. In contrast, with reference to Figure 12, as far as possible, the long shelves were alternated and interspersed with shorter shelves. This is so as to provide shoppers (agents) with as many available routes as possible so that they can disperse themselves (according to the avoidance coefficient as mentioned above), allowing for minimized human density around the items. These are two main reasons why our final layout proves superior to the given layout.

5.2.2.2 Independent variable: Modifying the shelves (changing their dimensions)

The following experiments / tests assume that the dimensions of the shelves can be changed and modified accordingly. A forced-path layout was attempted, shown below in Figure 14. However, this led to a high loss value of 51.49 as all customers were forced

down the same route, increasing density.

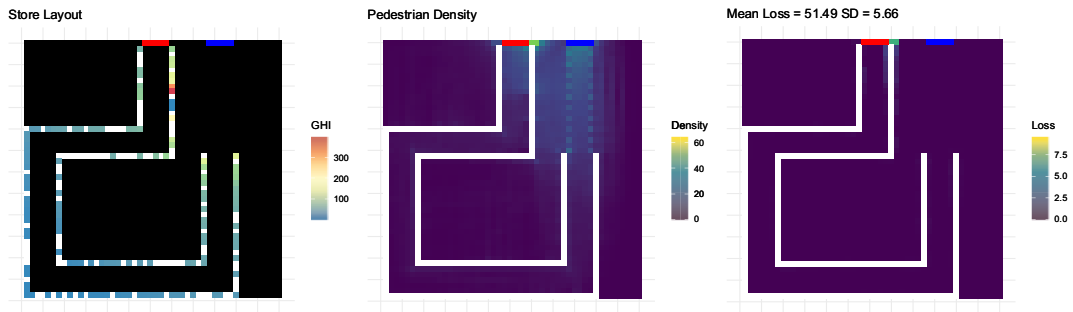


Figure 14: *Forced-path layout*

As such, this was improved by adding shortcuts between sections of the forced path without any goods, allowing customers with goods nearer the end of the forced path to avoid taking the main path, alleviating some of the density there. The simulation result is as shown in This significantly reduced the loss value from 51.49 to 29.59. However, this was still higher than the default. From this, it was theorised that a larger open space would allow for lower loss value. Hence, the forced-path layout was confirmed to be unsuitable and a more open concept was developed.

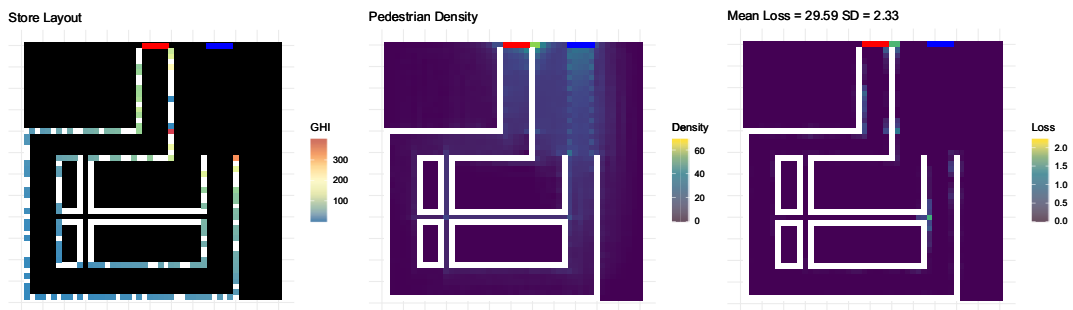


Figure 15: *forced path layout with shortcut*

The final experiment involved modifying the dimensions of the shelves to the point where each shelf only housed one item. Doing so allowed for the maximum space between shelves. The layout and arrangement of items on the shelves, the pedestrian density, as well as the loss value can be found in Figure 16

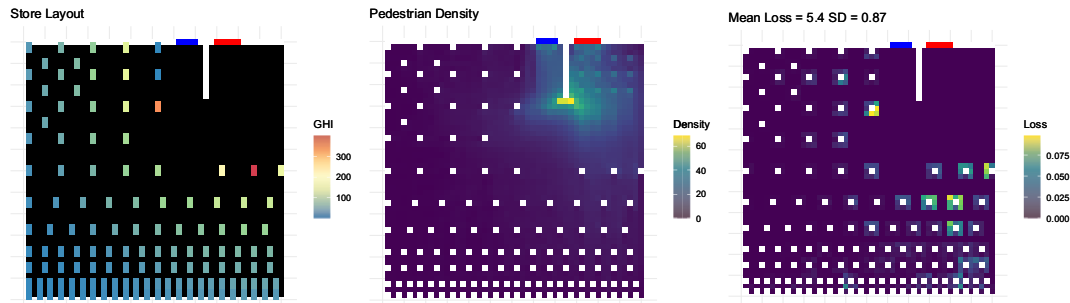


Figure 16: item arrangements, pedestrian density, and loss value diagrams for the 'open' layout.

With reference to Figure 16, it can be seen that for this layout, the shelves and items were arranged such that the items with high GHI were placed on shelves that were spread far apart, and items with low GHI were placed on shelves that were placed closer together. Thus, it prioritises minimizing human density around the shelves containing the high GHI goods. This strategy is shown to be highly effective, as evidenced by the pedestrian density simulation result, where most of the bitmap is dark blue in colour (low density). This is also evidenced by the mean loss result, where the mean monetary loss is only 5.4, the lowest result. Further iterations on this design were attempted, by selectively moving shelves with the highest loss, but no further improvements were achieved, indicating that this arrangement is close to the optimum possible.

6 Conclusions

The optimum item arrangement keeping the given layout is shown in Figure 5 (left side), with a loss of 17.99.

These shelves can be shifted around (while keeping the dimensions) to form the optimum layout with a loss of 9.71, as shown in Figure 12.

However, assuming modifying the dimensions of the shelves is allowed, it is possible to design a layout with even lower total loss of 5.4, as shown in Figure 16.

6.1 Recommendations

In this model, layouts were manually designed and tested using the simulation program. This was because testing each layout required considerable amounts of time; hence it was not feasible to use a randomizer program to come up with every possible layout combination and test every single one of them due to the sheer number of layout combinations. In future studies, given sufficient time and software resources, this approach can be made to make sure that the entire exhaustive list of possible layouts is tested, to come up with a definite optimum layout.

Additionally, in future studies, with more time resources, a larger number of studies can be consolidated to obtain more comprehensive statistics, such as the % of a particular

good getting damaged in shipping. This would allow for a more accurate determination and refining of the fragility index.

As mentioned above, due to the lack of reliable statistics, we also had to come up with a guess that 1000 people walking within 1 metre of an item on a busy day would cause the same amount of damage to the product as the product being shipped 10 times. In future studies, comprehensive analysis can be done on real-life customer behaviour, to more accurately determine how much damage would be done by 1000 customers walking within 1 metre of an item in 1 day. This was not done in this paper due to a lack of reliable sources. In addition, in this paper, we assumed that the agents, like shoppers in real-life, would tend to avoid crowded routes and use less-crowded routes instead to get to their destination faster. However, the extent to which shoppers in real-life exhibit this behaviour is unknown. In other words, in real-life, the precise ‘threshold’ of human density at which shoppers begin to utilize alternative routes is not known. Hence, the avoidance coefficient developed by our team might have been too large or too small. In future studies, more comprehensive research on shopper behaviour can be conducted to come up with a more accurate avoidance coefficient.

A similar issue was faced when coming up with the relationship between item damage and human density. As mentioned above, in this report, item damage was taken to be proportional to the square of the human density. Although it is a reasonable assumption that the amount of damage has a nonlinear relationship with density, the exact relationship is unknown. More analysis of consumer behaviour can be done in future studies to ensure the determination of a more accurate relationship. Nevertheless, the relationship between the amount of damage and human density can never be accurately determined, because as mentioned above, this is also influenced by subjective factors like the emotions and personalities of the shoppers.

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8 Letter to Store Manager

Dear Sir / Mdm,

Thank you for your continuous support to our team. Attached is our proposed layout for the upcoming 2020 Flash Sale.

After doing extensive research on the types of store layouts, we determined that the grid layout is the best choice for a flash sale. We determined that the factors affecting the popularity of a good are the customer rating, the discount amount, as well as the quantity of the good available. We took these into account to calculate the Gross Happiness Index (GHI), which measures the popularity of the good and hence the likelihood of the good being purchased by shoppers.

Armed with the above knowledge, our team tirelessly designed many grid layouts for the store. In designing these layouts, we followed various guidelines which we believed would lead to the lowest amount of damage to the items. For instance, shelves were arranged as spaciouly as possible to minimize human density; we placed the checkout area close to the exit to ensure that the customers who have paid for their goods would leave efficiently and not obstruct those who have not purchased their goods; and finally, based on the GHI of the goods, we arranged the goods in such a way that would lead to the least overall monetary loss.

We then built a computer program to test the layouts that we had designed. The program simulated the different routes that the shoppers would be able to take for the different layouts. The agents in the model simulate the behaviour of shoppers in real-life, such as how shoppers would avoid more crowded paths and utilize longer, but less crowded paths to reach their destination more quickly. We also managed to obtain data from reliable published sources to calculate the fragility of the different goods on sale. Thus, the final layout you see is the one that has come out on top through all the rounds of rigorous testing. This layout allows for the lowest human density around the most fragile and expensive items, reducing the overall expected monetary loss.

However, do note that our floor plan cannot prevent fights between shoppers, as this depends entirely on the personalities of the shoppers. Hence, please consider hiring security guards and installing security cameras or warning signs in the store to deter fights between shoppers.

We wish you the best of luck with your event.

Most Sincerely, The IMMC team