INTERNATIONAL MATHEMATICAL MODELING CHALLENGE

CONTEST RESULTS AND PAPER

IM^{2}C promotes the teaching of mathematical modeling and applications at all educational levels for all students. It is based on the firm belief that students and teachers need to experience the power of mathematics to help better understand, analyze and solve real world problems outside of mathematics itself – and to do so in realistic contexts. The Challenge has been established in the spirit of promoting educational change.

2019 IM^{2}C

The 5th Annual International Mathematical Challenge (IM^{2}C) culminated with the five Meritorious Teams presenting their solution at the ICTMA Conference in China Hong Kong (SAR) . Congratulations to our Meritorious teams and all teams participating in the 2019 IM^{2}C.

The IM^{2}C continues to be a rewarding experience for students, advisors, schools, and judges. A total of 57 teams, with up to 4 students each, representing 33 countries and regions competed this year.

The purpose of the IM^{2}C is to promote the teaching of mathematical modeling and applications at all educational levels for all students. It is based on the firm belief that students and teachers need to experience the underlying power of mathematics to help better understand, analyze, and solve real world problems outside of mathematics itself—and to do so in realistic contexts. The Challenge has been established in the spirit of promoting educational change.

For many years there has been an increased recognition of the importance of mathematical modeling from universities, government, and industry. Modeling courses have proliferated in under-graduate and graduate departments of mathematical sciences worldwide. Several university modeling competitions are flourishing. Yet at the school level, even amid signs of the growing recognitionof modeling's centrality, there are only a few such competitions with many fewer students participating. One important way to influence secondary school culture, and teaching and learning practices, is to offer a high-level presti-gious secondary-school contest that has both national and international recog-nition.With this in mind, we founded the International Mathematical Modeling Challenge (IM2C) in 2014 and launched the 1st annual Challenge in 2015.

The IM2C is a true team competition held over a number of days, with students able to use any inanimate resources. Real problems require a mix of different kinds of mathematics for their analysis and solution. And, real problems take time and teamwork. The IM^{2}C provides students with a deeper experience of how mathematics can explain our world, and the satisfaction of applying mathematics to a real world problem to develop a model and solution.

Plans for 2020

We invite countries to enter up to two teams, each with up to four students and one teacher/faculty advisor. The contest will begin in March and end in May. During that timeframe, teams will choose five (5) consecutive days to work together on the problem. The faculty advisor must then submit the paper and certify that students followed the contest rules.

The International Expert Panel will judge the papers in early June and will announce winners by late June. Papers will be designated as Outstanding, Meritorious, Honorable Mention, and Successful Participant with appropriate plaques and certificates given in the name of students, their advisor, and their schools.

In 2020 selected teams will present at ICME in Shanghai China. Complete information about IM^{2}C is at **www.immchallenge.org**

The 2019 IM^{2}C Problem

What is the Earth's *carrying capacity* for human life?

1. Identify and analyze the major factors that you consider crucial to limiting the Earth's carrying capacity for human life under current conditions.
2. Use mathematical modeling to determine the current carrying capacity of the Earth for human life under today's conditions and technology.
3. What can mankind realistically do to raise the carrying capacity of the Earth for human life in perceived or anticipated future conditions? What would those conditions be?

IM^{2}C Funding

Funding for planning and organizational activities is provided by IM^{2}C co-founders and co-sponsors: *Consortium for Mathematics and its Applications* (COMAP), a not-for-profit company dedicated to the improvement of mathematics education, and *NeoUnion ESC Organization* in China Hong Kong (SAR).

Note that IM^{2}C is aware of available resources and references that address and discuss this question. It is not sufficient to simply re-present any of these models or discussions, even if properly cited. Any successful paper must include development and analysis of your model.

Your submission should consist of:

- One-page Summary Sheet.
- Your solution of no more than 20 pages, for a maximum of 21 pages with your summary.
- A complete list of references with in-text citations.
- Note: Your reference list and any appendices do not count toward the 21-page limit and should appear after your completed solution.

Glossary

Carrying Capacity: The carrying capacity of a biological species in an environment is the maximum population size of the species that the environment can sustain indefinitely, given the food, habitat, water, and other necessities available in the environment.

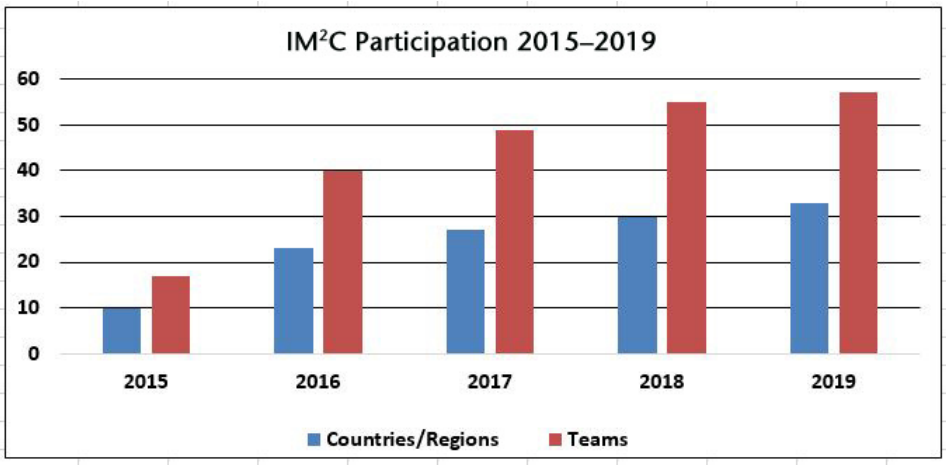
The 2019 IM^{2}C International Judges' Commentary

by Ruud Stolwijk

The IM^{2}C judges wish to congratulate all students who took part in the 2019 IM^{2}C. The judges were impressed by the efforts of all participating teams, the mathematics shown in the solutions, and the high quality of the final submissions. All of the 57 papers submitted (from 33 different countries/regions) showed great creativity in working on the Challenge. While the Expert Panel judges see only two papers from each participating country or region, we recognize that many more students participate in the Challenge. It is exciting to know that so many students are engaging in, and successfully completing, this mathematical modeling opportunity. We encourage students to continue to

The 2019 IM^{2}C Meritorious Teams

School, Location	Advisor	Team Members
St. Paul's Co-Educational College China Hong Kong (SAR)	Chan Lung Chak	Au Yee Fong Lam Justin Yuen Shing Ho Kinsey He Yixuan
Brisbane Boys College Toowong, Australia	Chicri Maksoud	James English Ethan Jonathan Waugh Xavier Catford Ometh Rajapakse
Manurewa High Manurewa, Auckland, New Zealand	Lawrence Naicker	Aimee Lew Ella Guiao Aaron Lew John Chen
Il Liceum Ogólnokształcące z Oddziałami Dwujęzycznymi Warszawa, Poland	Zbigniew Luchcinski	Adam Harrison Jakub Sliz Maksymilian Wolski Krzysztof Oldakowski
Utrechts Stedelijk Gymnasium Utrecht, The Netherlands	Gert Welleweerd	Ties Bloemen Lucas Baas Daniel Kunenborg Guido Siers



The 2019 IM^{2}C Expert Panel

Frank Giordano,
Naval Post Graduate School, USA – Chair

Konstantin K. Avilov,
Institute for Numerical Mathematics, Russia

Ruud Stolwijk,
Cito, The Netherlands

Liqiang Lu,
Fudan University, China

Jill Brown,
Australian Catholic University, Australia

Yang Wang,
The Hong Kong University of Science and Technology, China Hong Kong (SAR)

Henk van der Kooij,
Freudenthal Institute, The Netherlands

Kathleen Snook
COMAP, USA

form teams, compete to represent their country or region, and aspire to be named among the top teams at the international level Expert Panel judging.

This year's challenge was to develop a mathematical model to determine the Earth's carrying capacity for human life. This problem posed three requirements. First, students identified and analyzed the major crucial factors that currently limit the Earth's carrying capacity for human life. Next, teams developed and used a mathematical model to determine the current carrying capacity under today's conditions and technology. Finally, students addressed how mankind could raise the carrying capacity in perceived or anticipated future conditions.

This year's problem was quite a different type of problem than in the former four IM^{2}C in that it was fairly open ended. In many cases, teams found it difficult to find the appropriate mathematics to apply. Students performed analysis on data mined from available web sources, leading to a variety of descriptions of "the Earth's capacity for human life under today's conditions and technology," but not necessarily leading to a useful mathematical model to address the actual problem. Many teams concentrated their efforts on collecting, analyzing, and (sometimes) recalculating the data on annual supply rates and annual per capita demand rates of various resources. At that point, teams needed to analyze and look across these data, but many limited their mathematical model of carrying capacity to a very simple one that did not take into account that different critical resources are not independent. Thus, one of the main discriminators of "better papers" was inclusion of some interdependence of the resources into the mathematical model, as well as posing and solving some kind of optimization problem for such a model.

Characteristics of the better papers

From the 57 papers judged, five were considered to be Meritorious. The five countries/regions represented in this year's top category exemplify the growing international impact of IM^{2}C. We congratulate these top teams.

In these Meritorious papers, teams made good choices for the factors considered crucial. Drinkable water, food, energy, and space (both for living and food production) turned out to be critical factors for teams to include in their discussions. The explanation of the factors, and their interdependence, was quite crucial with all Meritorious teams doing a good job addressing this aspect of the problem. Also, these teams appropriately justified their choices and uses of particular information, facts, figures, and graphs found on the Internet. This is quite an important issue for the IM^{2}C judging: always justify the information you choose to use and what you do with it! Additionally, teams must document where they found their information by providing a reference to the source.

The beginning of a team's paper makes an important impression on the judges. A one-page summary at the start of the paper not only summarizes the team's work, but also serves as an invitation to the reader to read the rest of the paper. A good example of a nice summary can be found in paper 2019043 from Kamnoetvidya Science Academy (KVIS), Thailand.

The heart of the paper is in the modeling. An interesting modeling scheme was used in paper 2019032 from the Bayview Secondary School, Canada, where everything (including energy production and water supply via desalinization) was expressed in terms of land area used, with land size thus being the only limiting factor. In paper 2019038 from the Lauvyntyev Institute of Hydrodynamics, Russia, an optimization of human diet with respect to fats, carbohydrates, proteins, and energy consumption was used to determine the most efficient combination of crops to cultivate. As for choosing the limiting factors, there were several papers that also explored less obvious limiting factors like carbon emissions or non-recycled waste. Paper 2019018 from Brisbane Boy's College, Australia is a good example of a relatively short, well articulated, and straightforward presentation of the model.

Finally, the use of appendices is very suitable for both computer code and (big) data sets, since putting these items in appendices helps to increase the readability of the paper itself. Paper 2019017 from Escola Hou Kong, China Macau (SAR) provides a nice example of putting data used in the appendix. Please realize it is hardly likely the judges will read all computer code in the appendices, so the working of any computer program used must be succinctly, but fully, explained in the paper itself. Of course, a very important aspect in IM^{2}C is lucid and coherent explanation of the mathematical model that includes presentation of underlying concepts and assumptions. Teams must make decisions about what to include in the main body of the paper and what to include as supporting material in the appendices.

High-level sophisticated mathematics is not required to make a good model, however in this year's problem, as stated before, many teams had difficulty putting together an integrated

2019 Winning Papers

To view all the Meritorious papers visit

http://immchallenge.org/Contests/2019/Solutions.html

model. In some cases, in attempts to use higher-level mathematics, teams showed quite a lot of mathematics that were not related to the problem itself. Teams should make sure they use appropriate mathematics and that they explain their modeling processes and procedures. In this year's Challenge all models developed were more or less linear. Paper 2019025 from Maimonides School, Chile provides an example of linear programming, that is quite simple and direct, but well explained.

The judges were quite surprised that a number of papers found the Earth's carrying capacity for human life to be less than the present population of about 7.7 billion people. And, in some cases, the carrying capacity was stated to be significantly less. These papers offered little or no explanation of how or why this result was possible or accurate, or that the result was in any way remarkable given our current population. Models and solutions must always be thought of within the real-life relevance of the problem as IM^{2}C problems are about real-life scenarios.

Suggestions and advice for future participants

- As stated above, the summary is the reader's introduction to the paper. The summary must not only describe the way the problem is solved, it must also invite the reader to continue to read the full paper. The summary is not the place for complex mathematical descriptions, but instead a general introduction to the problem, your solution process, and your final conclusion.
- As teams only have a short time to do the Challenge they must make assumptions to focus and simplify their work. Teams should only make assumptions that actually impact their model and they should justify their assumptions.
- The most important part of a team's submission is their model. All other parts of their paper support the development, use, and analysis of the model. The mathematics used must always be explained in a logical manner, since this is the heart of modeling!
- Make sure as you model you always stay in touch with reality. Since the problem is a real-life problem, it is quite essential that teams reflect and critically judge their mathematical solution as calculated.
- Since even a good model cannot be perfect, especially when developed in only five days of work, an analysis of the strengths and weaknesses is required.
- Ensure that your paper concludes with a short summary of the actual solution or findings to the requirements of the problem.
- With only 20 pages to introduce the problem, state assumptions and justifications, develop, solve, and apply the model, and do some analysis to include identifying strengths and weaknesses, there is not much room for tables of data or code. While you might include a short section of code or a small subsection of a data spreadsheet, computer code and (big) data sets should not be in the paper itself but in an appendix.
- Teams must properly document and reference all information taken from the Internet or other sources, including graphs, illustrations, and pictures. Teams can use in-line documentation, footnotes, or endnotes and should also include a Reference List.
- Teams should take notice of the page limitation of 20 pages, and the rules for font size and margins.

The 2019 IM2C Meritorious Teams, their advisors, and contest officials.

Finally, the IM^{2}C judges would like to compliment all teams on their effort, and thank them and their teachers/advisors for participating in the 2019 IM^{2}C. All teams did a great job in diving into a complex problem for five days. The result was quite a number of very creative papers. The judges (all mathematicians and teachers) had some stimulating discussions about the papers, as it was clear that the students gave some very thought provoking analyses. This shows that in many countries and schools, mathematical modeling is a growing field of interest that students enjoy and are very capable of doing. Well Done!

General Advice to Teams Participating in Future IM^{2}C

The IM^{2}C is definitely a challenge. Teams have to organize themselves, address all requirements of the problem, and write a report all in a short period of time. Budgeting time becomes critical so that you leave enough time to effectively communicate your work and results to the Challenge judges.

Our advice is to allow plenty of time to construct your report. In fact, consider outlining the report as soon as you begin working on the problem. This outline will guide your team in its work and provide a logical path to your solution for readers of your report. Remember, you are communicating with judges from many countries of the world. The judges are not necessarily familiar with the curricula of your school, so present the development of your model in a logical and easily understood fashion. Judges are not looking for the papers that use the most sophisticated mathematics. Do not force the mathematics upon a given scenario. Rather, begin with the simplest mathematics that solves the problem you have identified. Later, as appropriate, refine and enhance your model to increase its precision, or adjust your assumptions to find a more broadly appropriate solution.

Pictures, graphs, tables, and schedules can be quite effective and efficient in communicating your ideas. The use of relevant pictures and graphs can make a report clearer and more pleasant to read. Your report should include a combination of various representations: symbolic, graphical, and text that best present your model and solution. Realize, however, that large tables and extensive code or data might be better as supporting material located in an appendix.

The use of symbolic formulae and algorithms are quite essential in a mathematical modeling assignment. The use of unexplained formulae, however, will not make the report more convincing. The reader needs to believe that the writers themselves understand the formulae used. This is done through explanations and analyses of your modeling processes. The readers of your report, while experienced mathematicians, are not experts in all parts of the great world of math!

Appendices are very useful, but do not expect the judges to read them. While judges may refer to an appendix to check a reference or to get a general idea of your computer code, they will not fully read the appendices. Therefore, do not place anything critically important to the development of your model in an appendix.

Remember to list any sources you used during your work on the Challenge and to document in your paper where you used these sources (e.g. a graph or picture from a particular web site). Follow the rules of completing your solution report within the specified number of pages and in a font size of no smaller than 12 point type.

Overall, present the development and analysis of your model in a manner that a wide audience could understand. Consider who might be using your model and explain your model to that audience, as well as the judges. Ensure you close your report with a conclusion and a summary of your results.

For more information about the IM^{2}C, including the complete 2015–2019 results and sample papers, visit

www.immchallenge.org